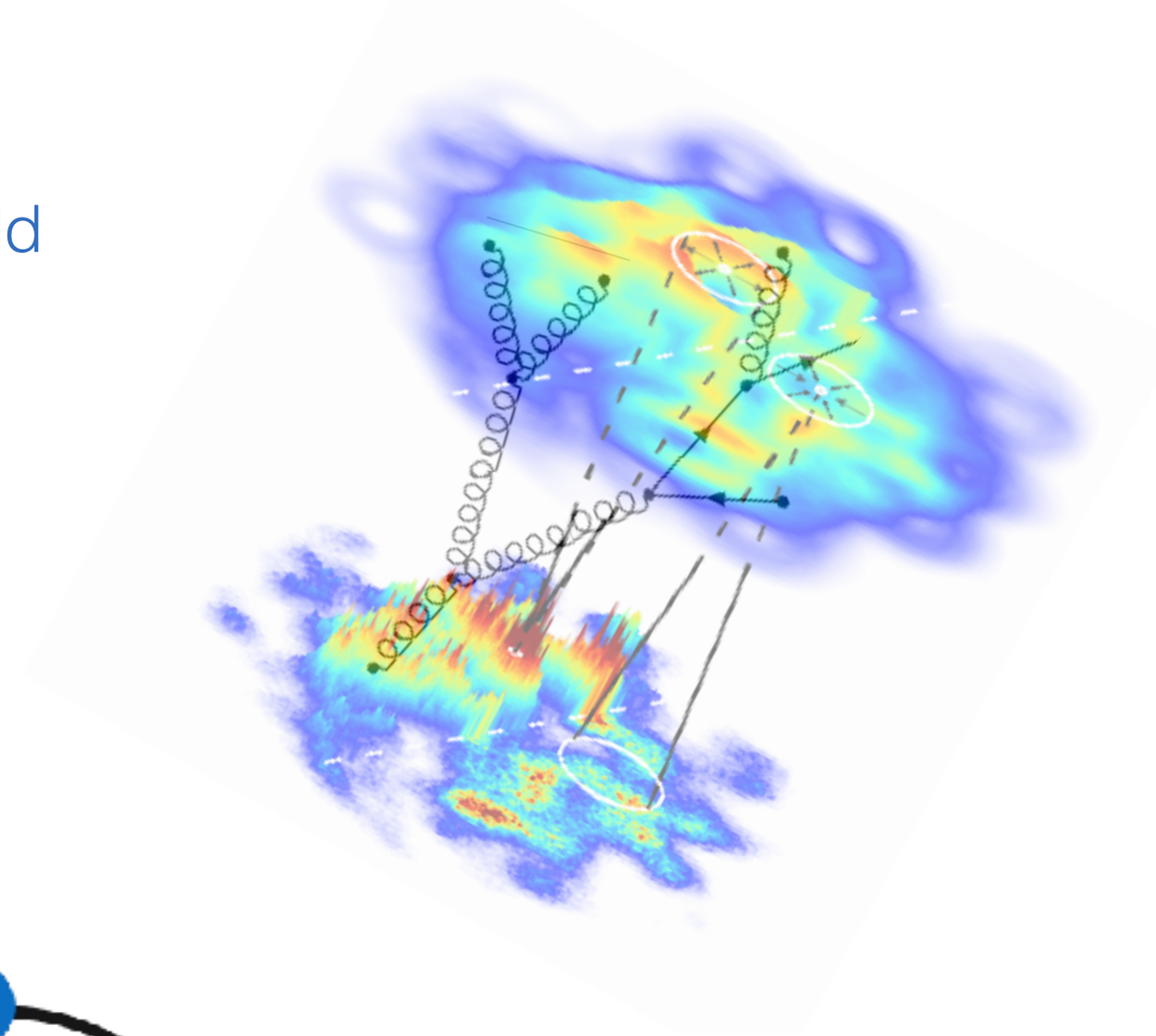
Hydrodynamic attractors, entropy production and initial state energy in relativistic Heavy-Ion Collisions

Sören Schlichting | Universität Bielefeld

Based on
Giacalone, Mazeliauskas, SS
Phys.Rev.Lett. 123 (2019) 26, 262301



S@INT Online Seminar

INT, Seattle Apr 2020

CRC-TR 211
Strong-interaction matter under extreme conditions

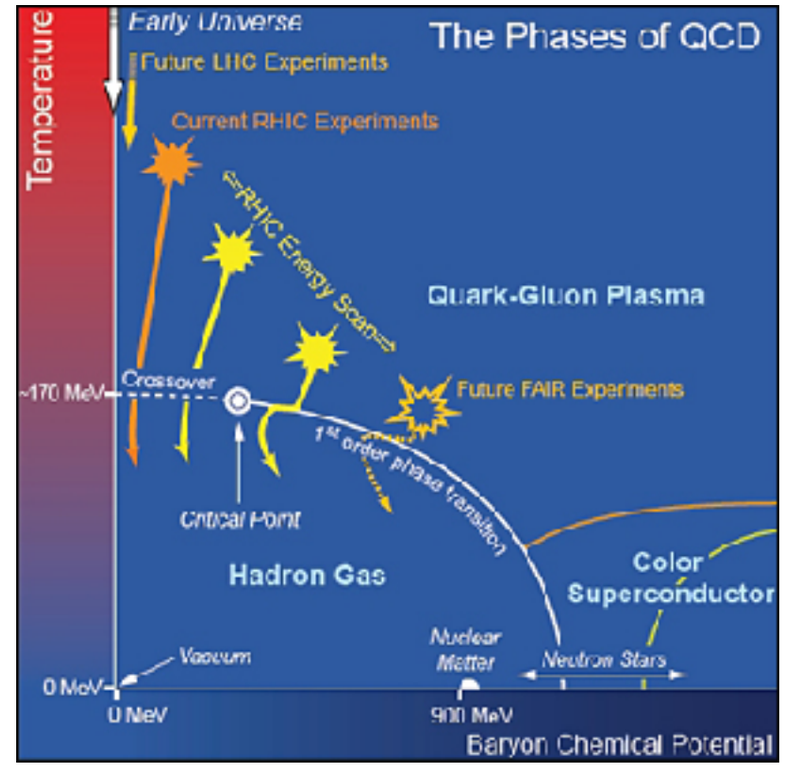
UNIVERSITÄT BIELEFELD

Overview

1. Motivation & Introduction
2. Early time dynamics & entropy production in high-energy Heavy-Ion Collisions
3. Basic phenomenology
4. Summary & future directions

High energy Heavy-Ion Collisions

Science goal is to explore the structure and dynamics of the strong interaction matter (Quarks & Gluons) described by QCD



Normal conditions: Quarks & gluons confined in color neutral bounds states (π,K,p,n,,…)

Extreme conditions (high resolution, high temperature): allow to explore fundamental constituents

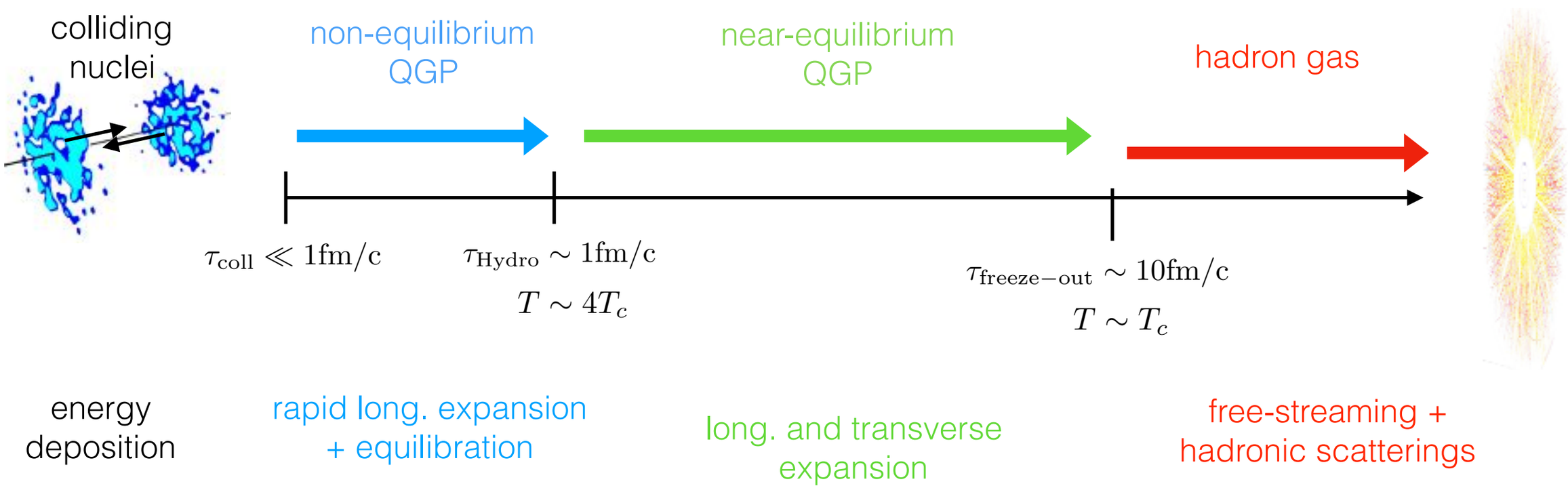
Explored in high-energy Heavy-Ion collisions experiments at RHIC (BNL) and LHC (CERN) as well as GSI/FAIR (Darmstadt) and NICA (Dubna)

Extraction of QCD properties from heavy-ion experiments requires profound understanding of the space-time dynamics of the collision

Dynamics of HICs

Dynamical description of heavy-ion collisions from underlying theory of QCD remains an outstanding challenge

Standard description of nucleus-nucleus (A+A) collisions based on separation of time scales in the reaction dynamics

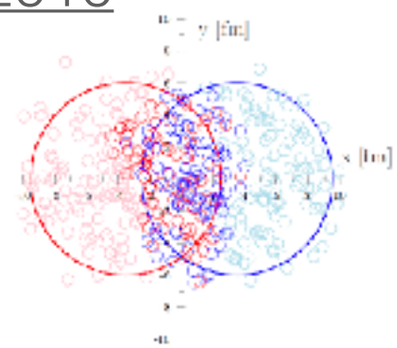


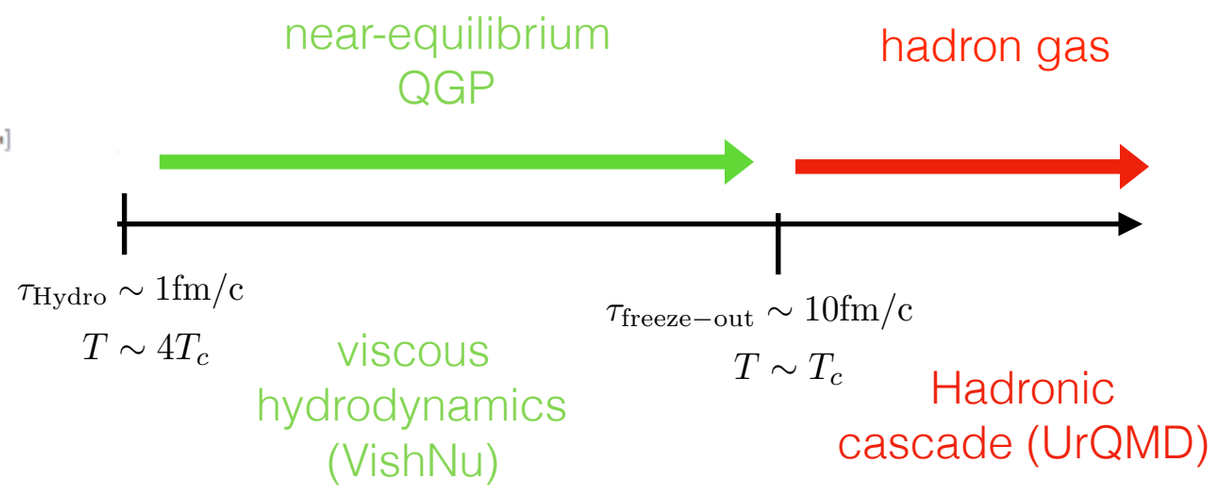
Theoretical description based on multi-stage evolution models

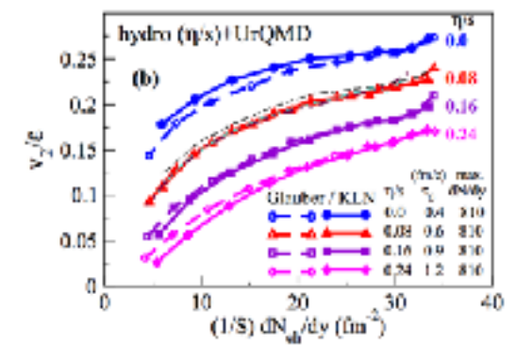
Multi-stage models of HICs

Status ~2010

Song et el. PRL 106 (2011) 192301



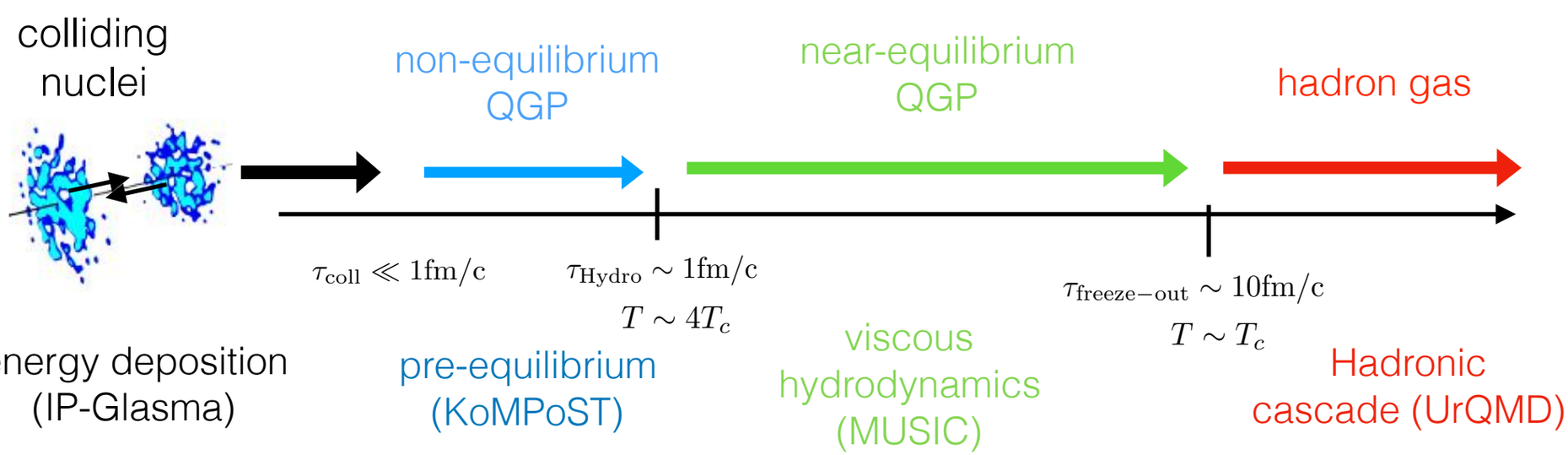


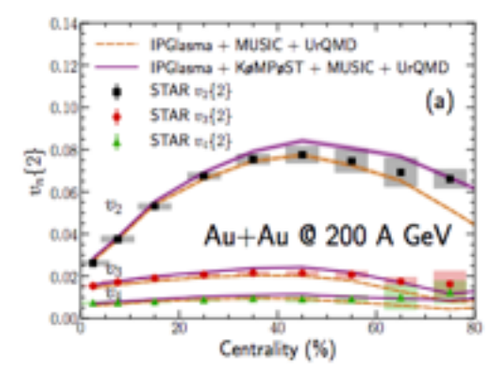


Experimental comparison

Current state-of-the-art

Gale et el. arXiv:2002.05191



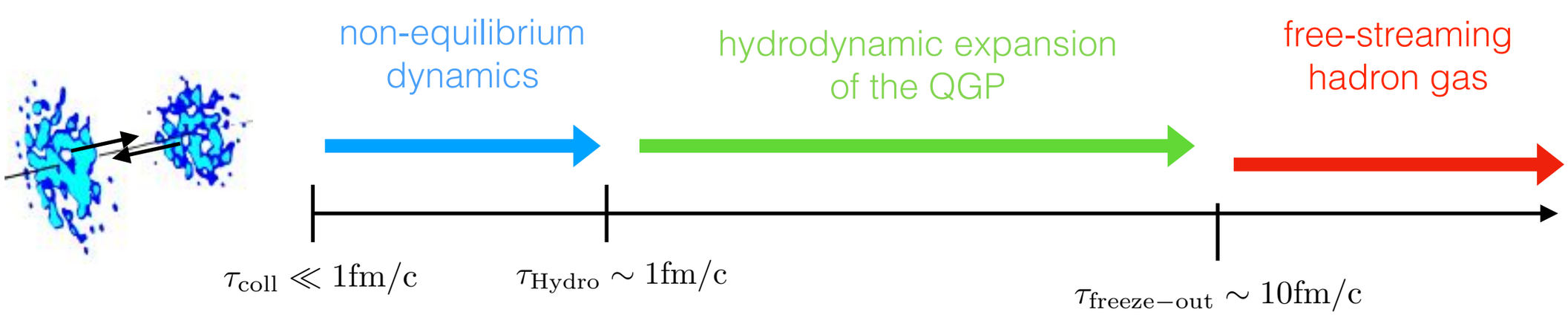


Statistical analysis

Significant progress in theoretical description and experimental analysis over past 10 years have lead to better understanding of QGP properties

Dynamics of HICs

Space-time dynamics of HICs dominated by near-equilibrium hydrodynamic expansion



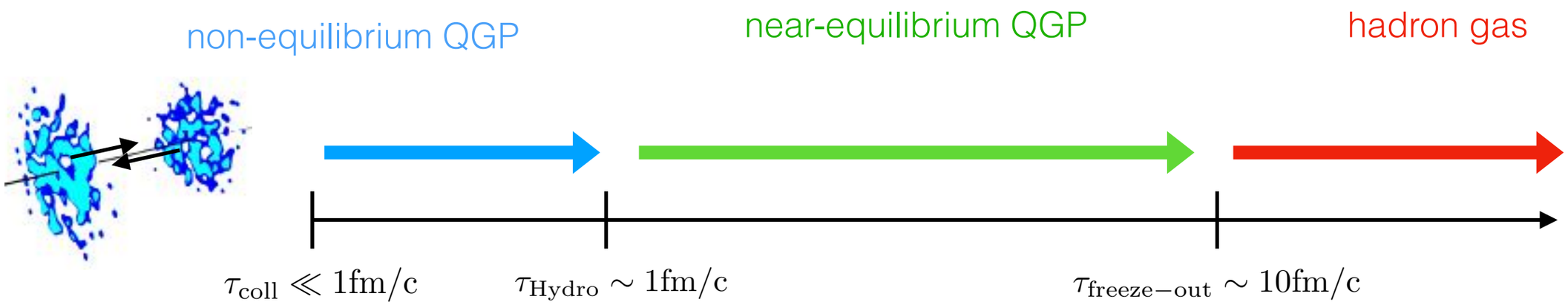
Small effects of pre-equilibrium dynamics on typical observables (v_n,<p_T>,…) dominated by transverse expansion of QGP (~1-10fm/c)

Controlled extraction of near-equilibrium and transport properties of QGP (EoS,η/s,..) from statistical comparison of theory/experiment based on hydrodynamic simulations of heavy-ion collisions

Difficult to gain access to non-equilibrium features in from experimental measurements in nucleus-nucleus collisions

Sensitivity to Initial state

Entropy production occurs only when system is significantly out-of-equilibrium



Entropy production in HICs is dominated by early time dynamics and directly accessible by measurement of $dN_{ch}/d\eta$

Schematically:

(nearly) isentropic expansion

$$\left\langle \frac{dE_\perp}{d\eta} \right\rangle_{\tau_{\rm coll}} \longrightarrow \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\rm Hydro}} \longrightarrow \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\rm freeze-out}} \approx \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\rm Hydro}} \longrightarrow \left\langle \frac{dN_{\rm ch}}{d\eta} \right\rangle \approx \left\langle \frac{S}{N_{\rm ch}} \right\rangle \left\langle \frac{dS}{d\eta} \right\rangle_{\tau_{\rm freeze-out}}$$

initial entropy production

freeze-out

Based on insights from non-equilibrium studies, can now make relation between $dE/d\eta$ and $dN_{ch}/d\eta$ explicit

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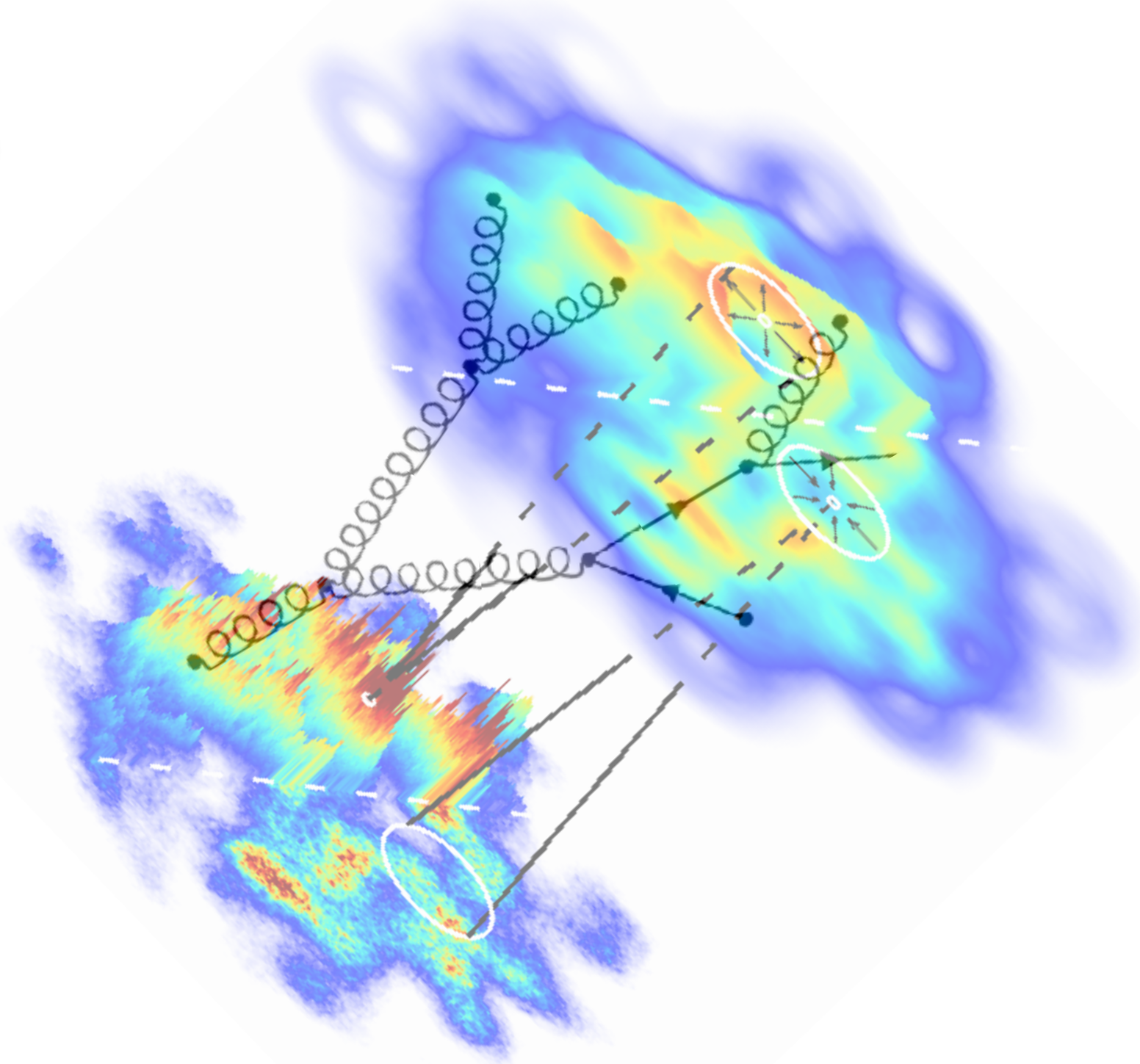
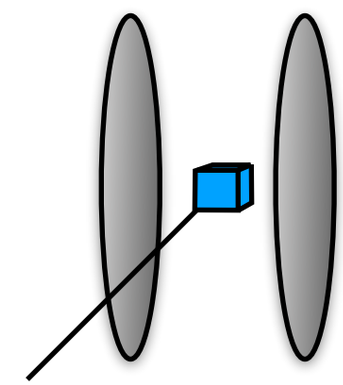


Early time dynamics &
entropy production

Early time dynamics & entropy production

Non-equilibrium initial state better characterized in terms of energy density (e), better to use entropy density (s) only once system is close to equilibrium

Generally interested in evolution over short time scales $\tau \sim 1\text{fm}/c \ll R_A$ where long. expansion is rapid ($\sim 1/\tau$) but transverse dynamics ($\sim 1/R$) can be neglected



Energy momentum tensor assumes average form $T^{\mu\nu} = \mathrm{diag}(e, p_T, p_T, p_L)$

Evolution of energy density is governed by conservation equation

$$\partial_\tau \epsilon = -\frac{\epsilon}{\tau} - \frac{P_L}{\tau}$$

Need one additional relation $P_L(e,\tau)$ to obtain evolution

Early time dynamics & entropy production

Equilibrium: Equation of state

$$\frac{P_L}{\epsilon} = \frac{1}{3}$$

Near-equilibrium:

Hydrodynamic constitutive relations based on grad. expansion

$$\frac{P_L}{\epsilon} = \frac{1}{3} - \frac{16}{9}\frac{\eta/s}{T(\tau)\tau}$$

Far-from equilibrium:

Generally no constitutive equations and P_L/e depends on microscopics

Need to investigate within different microscopic descriptions

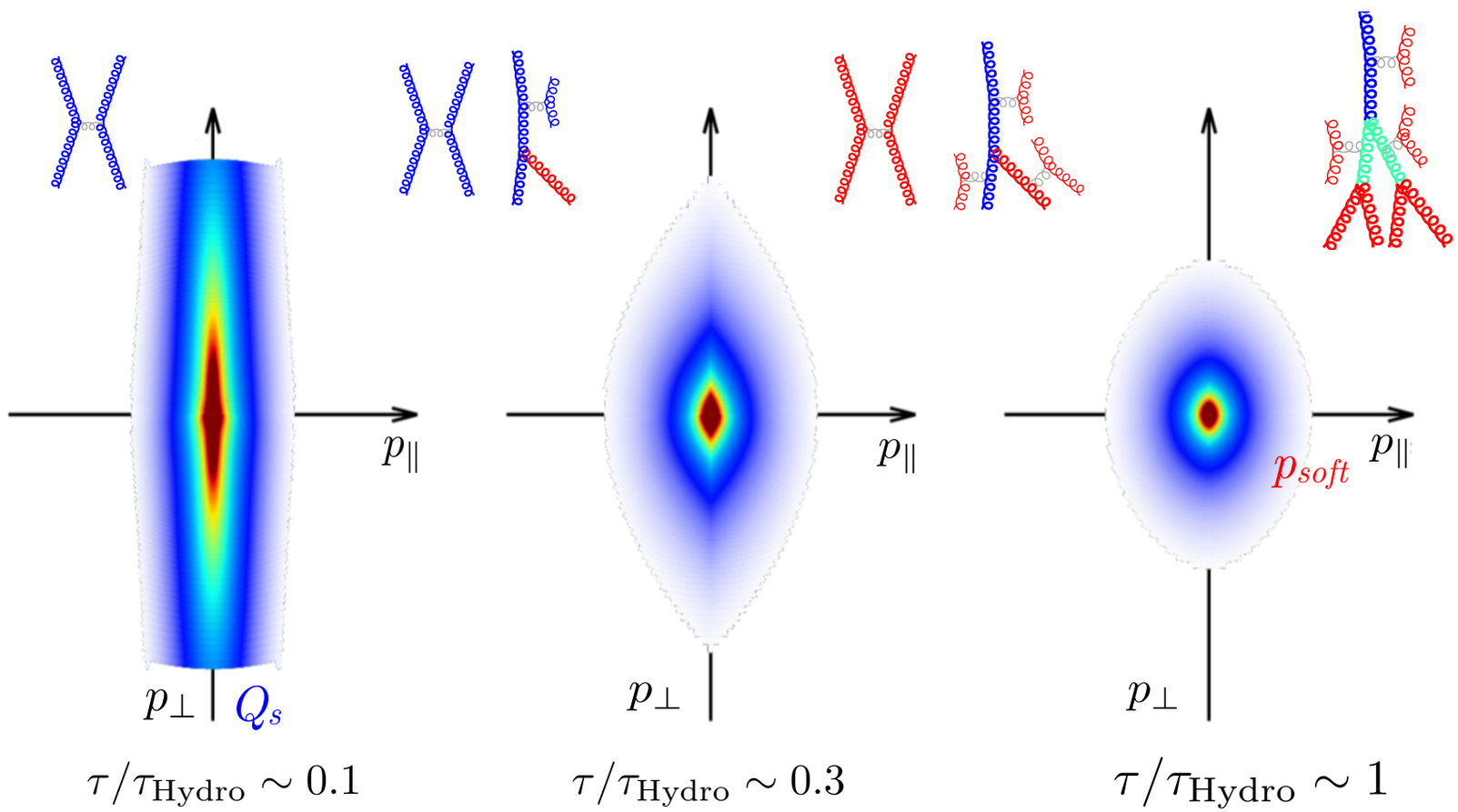
QCD/YM Kinetic theory, AdS/CFT, Boltzmann RTA

Microscopic evolution

Evolution of homogenous boost invariant system in QCD kinetic theory

Kurkela, Zhu PRL 115 (2015) 182301; Keegan,Kurkela,Mazeliauskas,Teaney JHEP 1608 (2016) 171;
Kurkela, Mazeliauskas, Paquet, SS, Teaney PRL 122 (2019) no.12, 122302; PRC 99 (2019) no.3, 034910

$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{2\leftrightarrow 2}[f] + C_{1\leftrightarrow 2}[f]$$



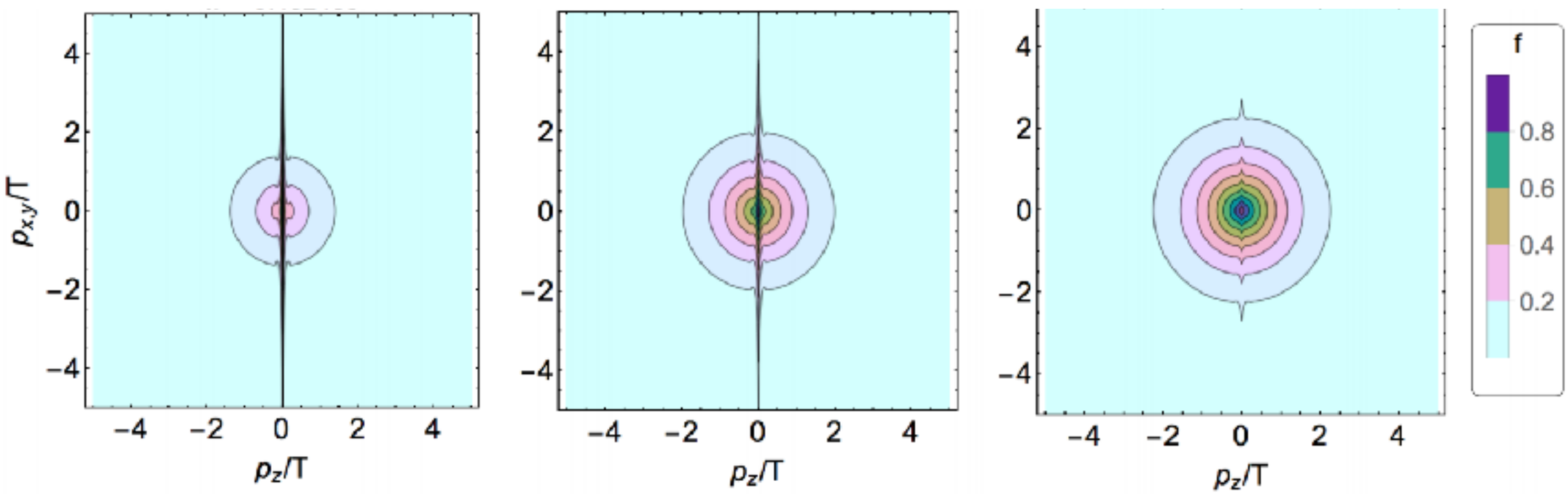
Kinetic equilibration ``bottom-up" via radiative break-up

Microscopic evolution

Evolution of homogenous boost invariant system in Boltzmann RTA

M. Strickland JHEP 1812 (2018) 128

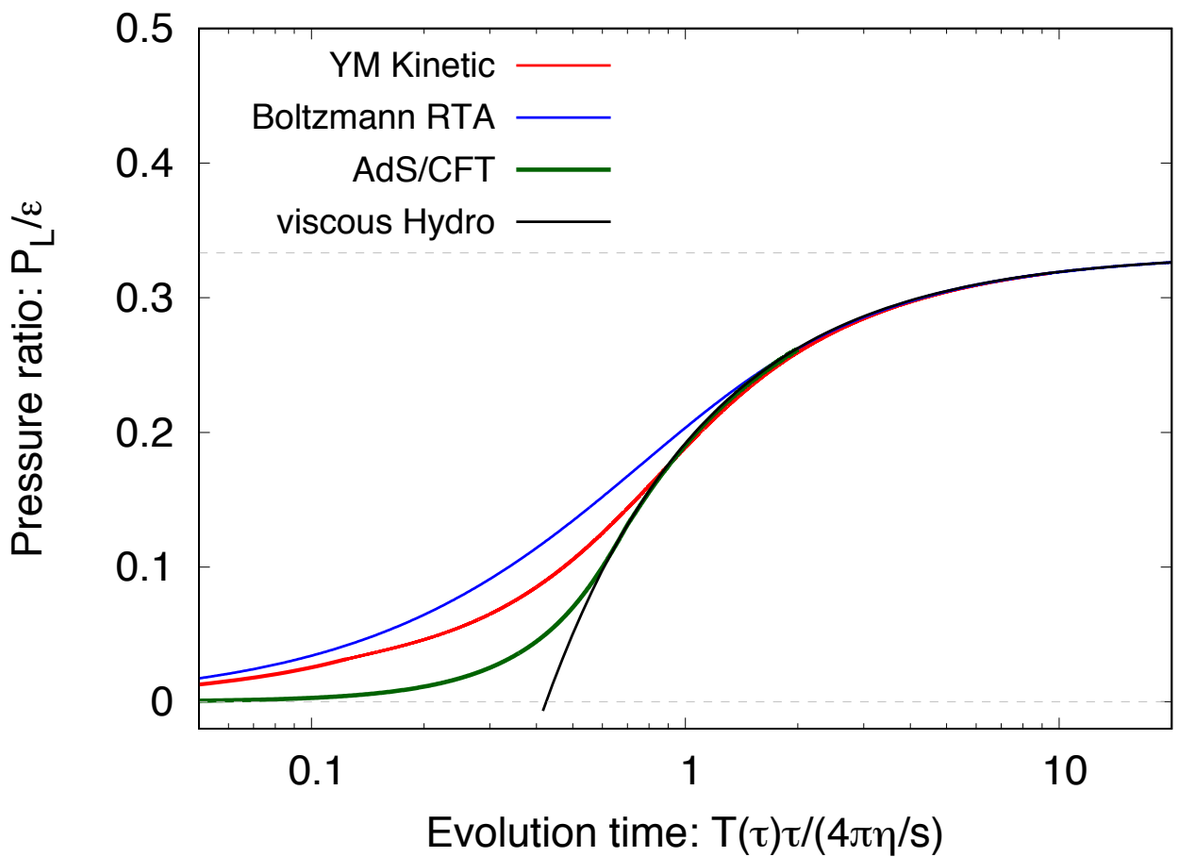
$$p^\mu \partial_\mu f(x,p) = -\frac{p^\mu u_\mu(x)}{\tau_R(x)}\Big[f(x,p) - f_{\rm eq}(x,p)\Big]$$



$\tau/\tau_{\rm Hydro} \sim 0.16$ $\tau/\tau_{\rm Hydro} \sim 0.4$ $\tau/\tau_{\rm Hydro} \sim 0.9$

Energy & pressure evolution

Despite clear microscopic differences the macroscopic features of the evolution are remarkably similar when compared in meaningful fashion



Viscous hydrodynamics becomes applicable on time scales

$$\tau/\tau_R^{\rm eq}(\tau) \approx 1 \qquad \tau_R^{\rm eq}(\tau) = \frac{4\pi\eta/s}{T_{\rm eff}(\tau)}$$

when Knudsen number Kn~1 and system is out-of-equilibrium

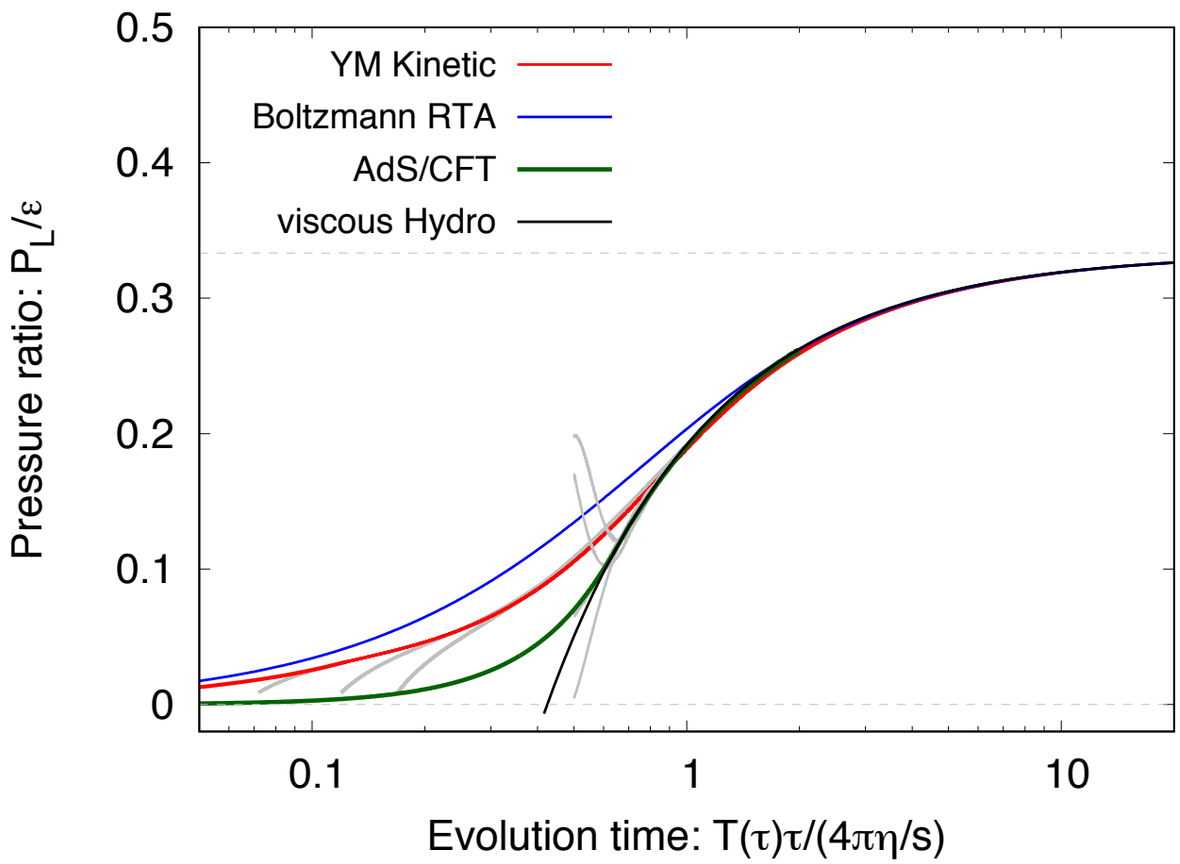
Hydrodynamization (≠ kinetic thermalization) time in A+A collisions

$$\tau_{\rm hydro} \approx 1.1\ {\rm fm} \left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}} \left(\frac{\langle \tau s\rangle}{4.1\ {\rm GeV}^2}\right)^{-1/2}$$

YM: Kurkela, Mazeliauskas, Paquet, SS, Teaney PRL 122 (2019) no.12, 122302; PRC 99 (2019) no.3, 034910
QCD: Kurkela, Mazeliauskas PRL 122 (2019) 142301; RTA:Kamata, Martinez, SS in prep. AdS/CFT: Romatschke PRL 120 (2018) no.1, 012301

Hydrodynamic attractors

Effective memory loss occurs at very early times $\tau << \tau_{Hydro}$
where long. expansion dominates the dynamics



Non-equilibrium evolution towards hydrodynamics described by "hydrodynamic attractor"

Heller, Spalinski PRL 115 (2015) no.7, 072501

Effective constitutive relations for far-from-equilibrium systems

$$\frac{P_L}{\epsilon} = f(\epsilon, \tau) = f\left(\tilde{w} = \frac{T(\tau)\tau}{4\pi\eta/s}\right)$$

Different microscopic theories have different attractors

Universality at early times (free-streaming)
& late times (visc. hydrodynamics)

Hydrodynamic attractors

Effective constitutive equation allows to obtain evolution of energy density from

conservation law

$$\partial_\tau \epsilon = -\frac{\epsilon}{\tau} - \frac{P_L}{\tau}$$

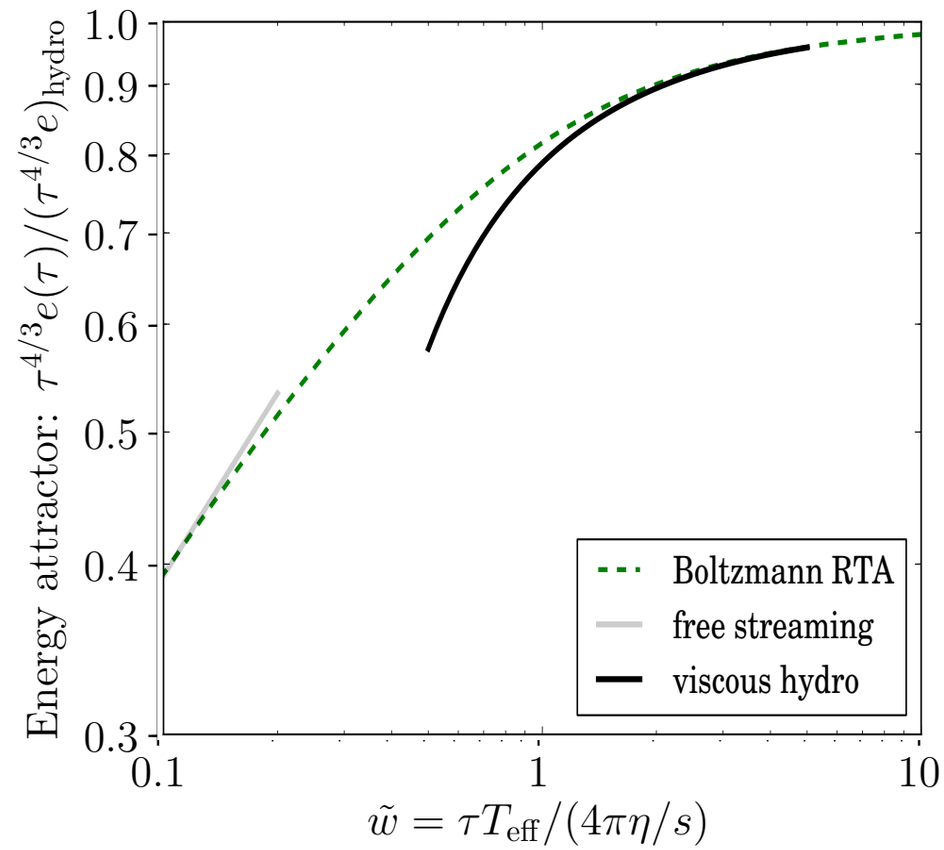
+

eff. constitutive relation

$$\frac{P_L}{\epsilon} = f\left(\tilde{w} = \frac{T(\tau)\tau}{4\pi\eta/s}\right)$$

yielding

$$e(\tilde{w}_\tau) = e(\tilde{w}_0)\exp\left(-\int_{\tilde{w}_0}^{\tilde{w}_\tau} \frac{d\tilde{w}}{\tilde{w}} \frac{1 + P_L(\tilde{w})/e(\tilde{w})}{\frac{3}{4} - \frac{1}{4}P_L(\tilde{w})/e(\tilde{w})}\right)$$



conveniently characterized by energy attractor function

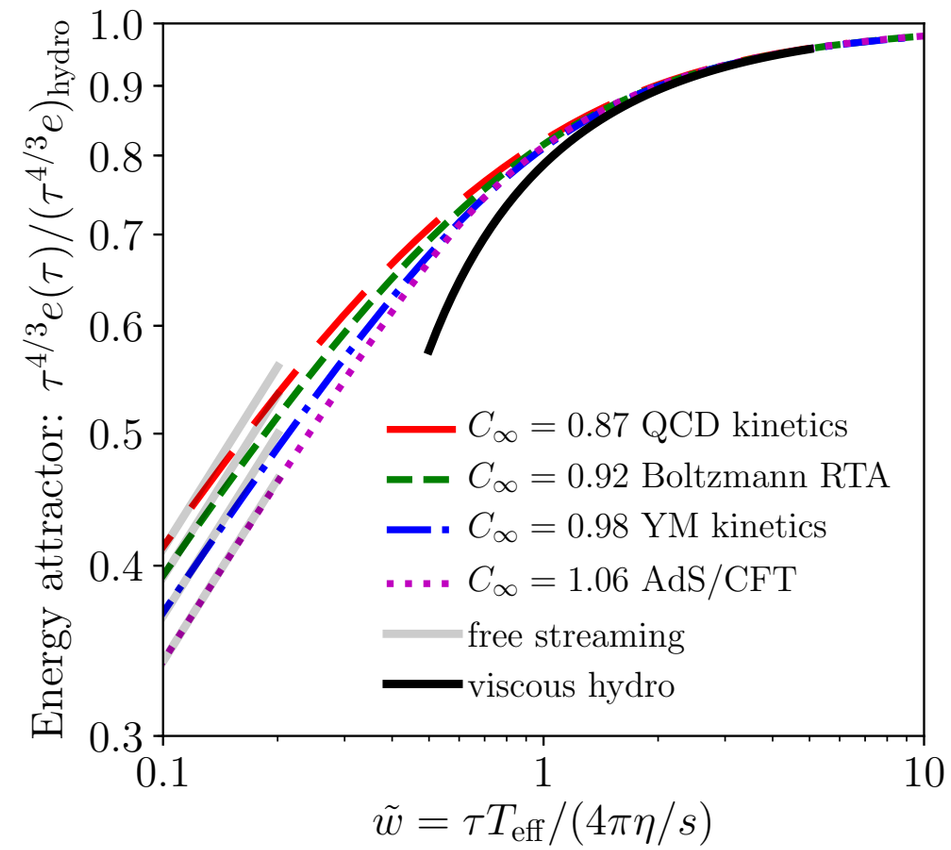
$$\frac{e(\tau)\tau^{4/3}}{e_{\rm hydro}\tau_{\rm hydro}^{4/3}} = \mathcal{E}\left(\tilde{w} = \frac{T_{\rm eff}(\tau)\tau}{4\pi\eta/s}\right)$$

Giacalone, Mazeliauskas, SS PRL 123 (2019) 26, 262301

Hydrodynamic attractors

Evolution of energy-density during pre-equilibrium phase described by

$$\frac{e(\tau)\tau^{4/3}}{e_{\rm hydro}\tau_{\rm hydro}^{4/3}} = \mathcal{E}\left(\tilde{w} = \frac{T_{\rm eff}(\tau)\tau}{4\pi\eta/s}\right)$$



Universal characteristics at early and late times

$\tilde{w} \ll 1$ macroscopically free-streaming ($P_L/e \approx 0$)

$$\mathcal{E}(\tilde{w} \ll 1) = C_\infty^{-1}\tilde{w}^{4/9}$$

$\tilde{w} \gg 1$ viscous hydrodynamics ($P_L/e \approx 1/3$- visc. correction)

$$\mathcal{E}(\tilde{w} \gg 1) = 1 - \frac{2}{3\pi\tilde{w}}$$

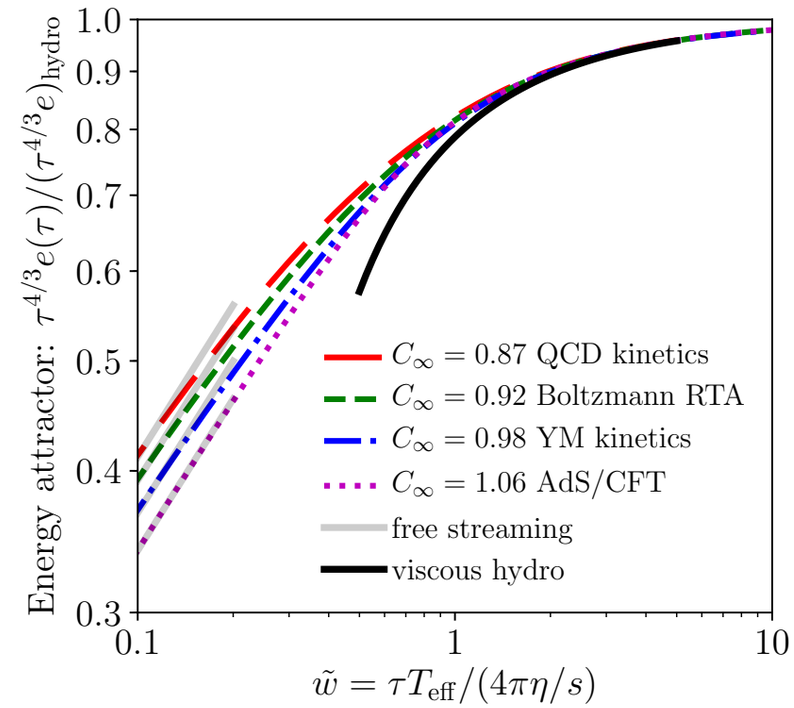
Surprisingly small differences between microscopic theories

Giacalone, Mazeliauskas, SS PRL 123 (2019) 26, 262301

Entropy production in HICs

Based on hydrodynamic attractor curve for energy density

$$\frac{e(\tau)\tau^{4/3}}{e_{\rm hydro}\tau_{\rm hydro}^{4/3}} = \mathcal{E}\left(\tilde{w} = \frac{T_{\rm eff}(\tau)\tau}{4\pi\eta/s}\right)$$



can use thermodynamic relations s=(e+p)/T and EOS once QGP is close to equilibrium to calculate entropy

$$(s\tau)_{\rm hydro} = \frac{4}{3}C_\infty^{3/4}\left(4\pi\frac{\eta}{s}\right)^{1/3}\left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{1/3}(e\tau)_0^{2/3}$$

Since overall entropy is approx. conserved during hydro expansion charged particle multiplicity at freeze-out directly determined

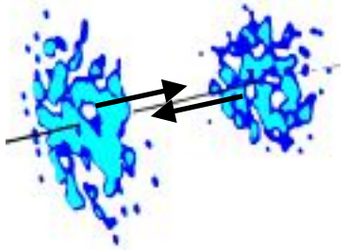
$$\frac{dN_{\rm ch}}{d\eta} \approx A_\perp\,(s\tau)_{\rm hydro}\frac{N_{\rm ch}}{S}$$

Giacalone, Mazeliauskas, SS PRL 123 (2019) 26, 262301

Entropy production in HICs

Based on macroscopic considerations one can establish one-to-one correspondence between initial state energy density and charged particle multiplicity including all relevant pre-factors

$$\frac{dN_{\rm ch}}{d\eta} \approx \frac{4}{3}\left(\frac{N_{\rm ch}}{S}\right) A_\perp C_\infty^{3/4}\left(4\pi\frac{\eta}{s}\right)^{1/3}\left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{1/3}(\epsilon\tau)_0^{2/3}$$



<u>Sensitivities/Uncertainties:</u>

Equilibrium properties: $N_{ch}/S \sim 7.5$, $\nu_{eff} \sim 40$ approximately known

Non-equilibrium/transport properties:
$C_\infty \sim 0.95 \pm 0.15$ surprisingly well constraint
$4\pi\, \eta/s \sim (1\text{-}3)$ not well constraint in relevant temperature range ($T \sim 4T_c$)

Initial state energy density:
$(e\tau)_0$ significant uncertainties from small-x TMDs and perturbative corrections

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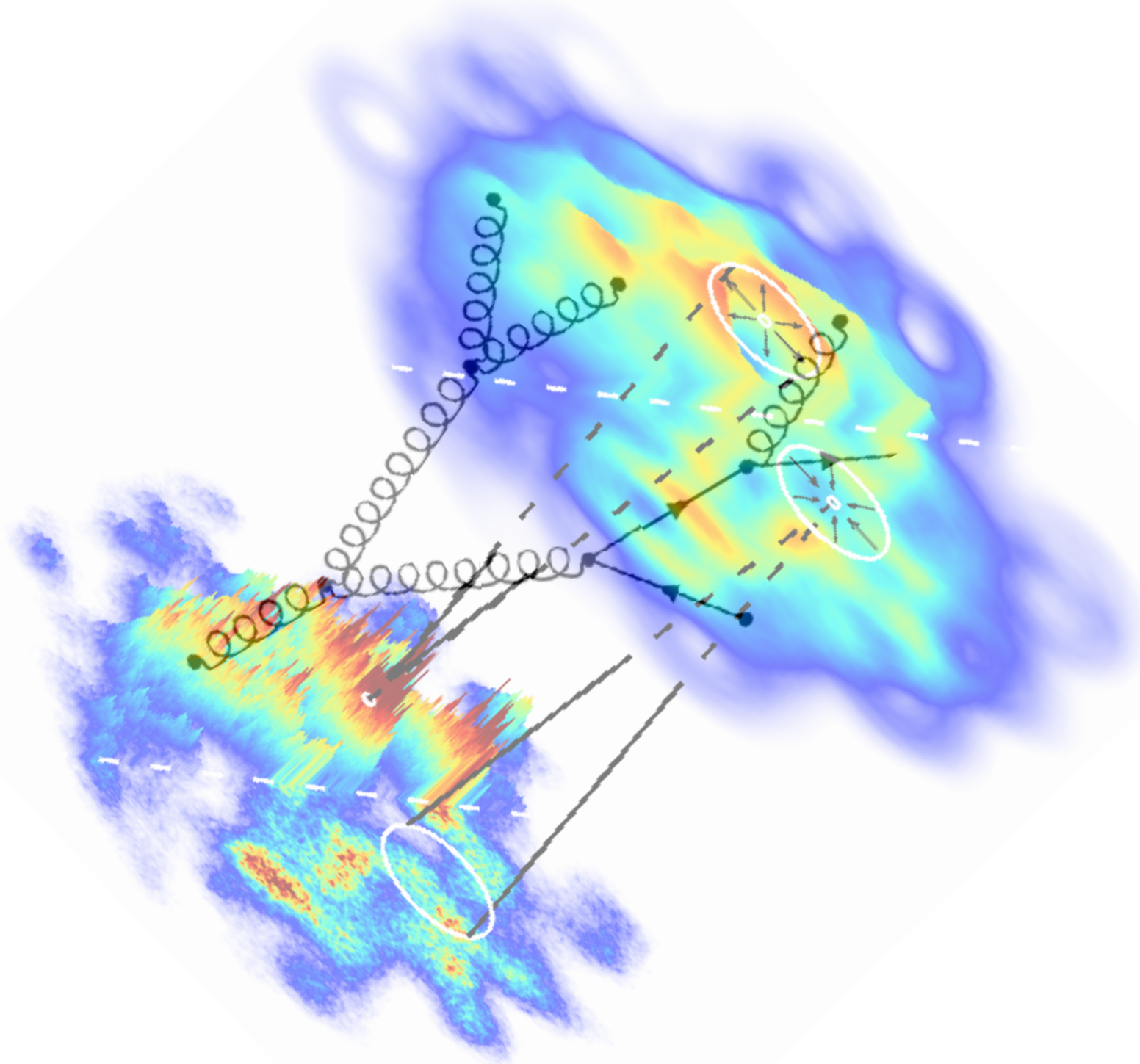


Basic phenomenology

Estimating magnitude of effects

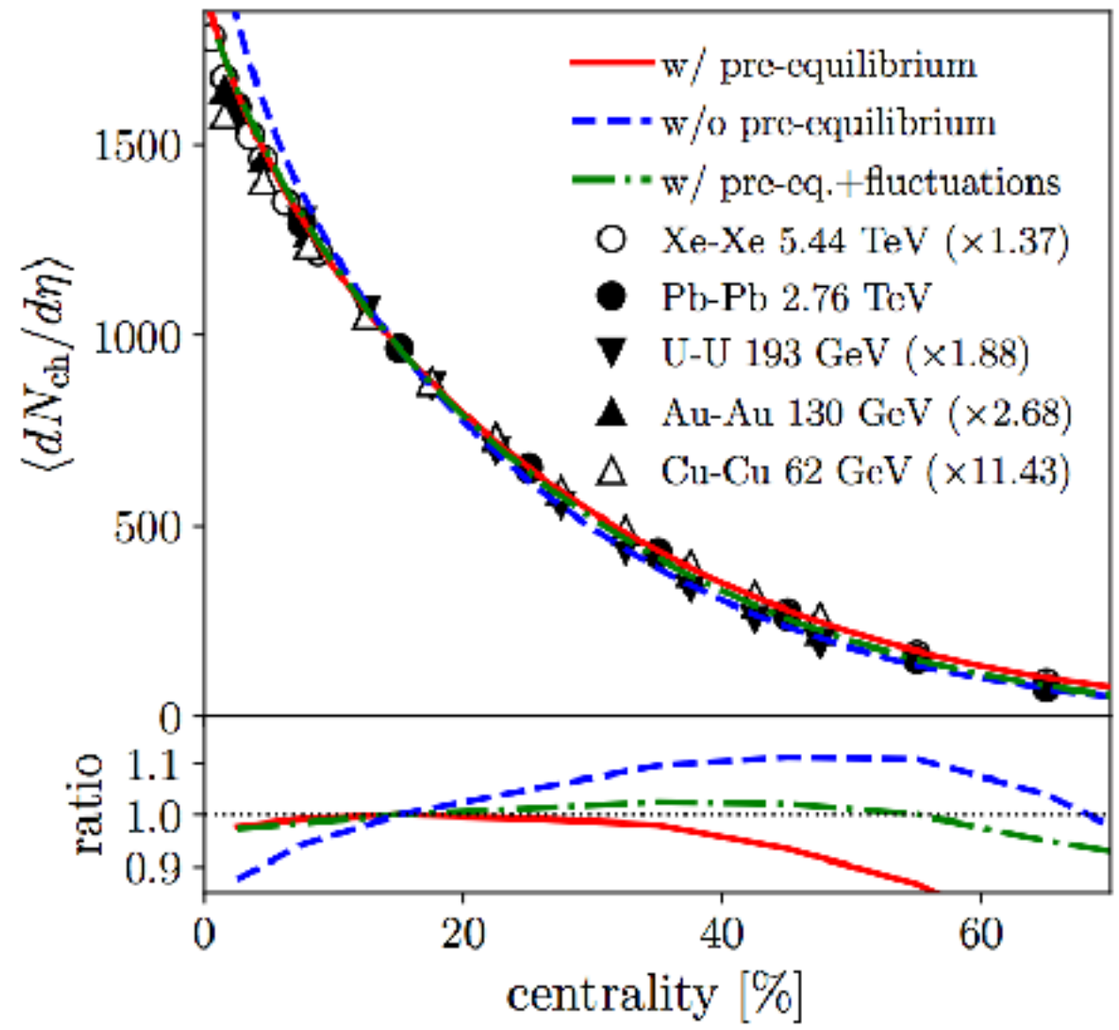
Estimate pre-equilibrium effects within simplistic saturation model

Initial state scenario:
No entropy production
$N_{ch} \sim N_{gluon}$

$$(n\tau)_0(\mathbf{x}_\perp) \propto (Q_s^<)^2(\mathbf{x}_\perp),$$

Thermalized scenario:
Pre-eq entropy production
$N_{ch} \sim S_{Hydro}$

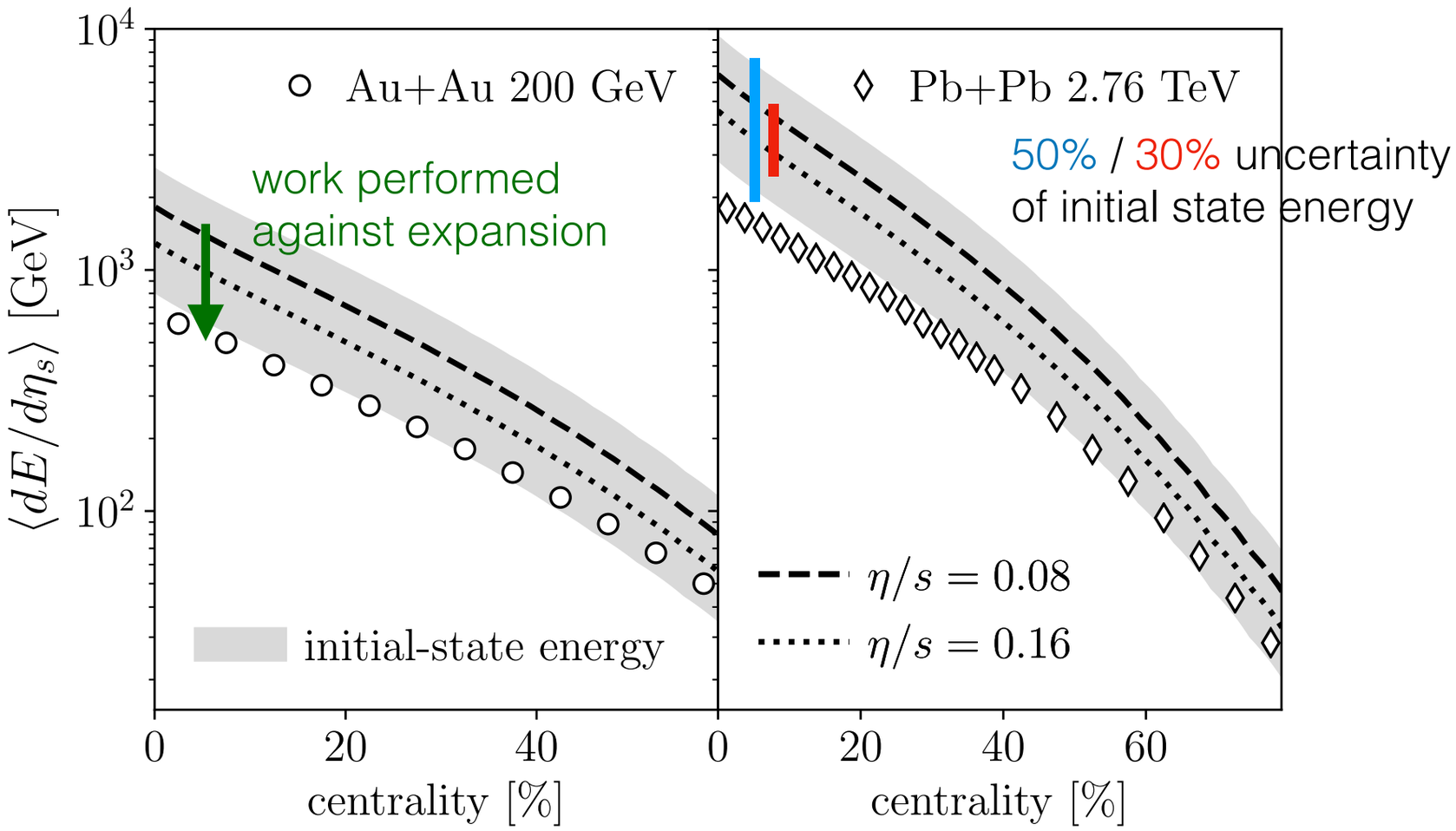
$$(e\tau)_0(\mathbf{x}_\perp) \propto (Q_s^<)^2(\mathbf{x}_\perp)Q_s^>(\mathbf{x}_\perp)$$



Sensitivity for centrality dependence only on the few percent level, mainly due to the fact that normalization is adjusted to reproduce data

Initial state in HICs

Can reverse formula to re-construct initial energy per rapidity



Sensitivity to η/s and $(e\tau)_0$ can be exploited to obtain combined constraints on initial state & transport properties

Giacalone, Mazeliauskas, SS PRL 123 (2019) 26, 262301

Conclusions & Outlook

Entropy production in HICs dominated by early time pre-equilibrium dynamics

Based on hydro attractors we can connect properties of initial state with experimental measurements

modern "Bjorken Formula":

$$\frac{dN_{\rm ch}}{d\eta} \approx \frac{4}{3}\left(\frac{N_{\rm ch}}{S}\right) A_\perp C_\infty^{3/4} \left(4\pi\frac{\eta}{s}\right)^{1/3} \left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{1/3} (\epsilon\tau)_0^{2/3}$$

Giacalone, Mazeliauskas, SS PRL 123 (2019) 26, 262301

Detailed study based on microscopic initial state models and KoMPoST* (pre-equilibrium computer code including transverse dynamics) in progress *Kurkela, Mazeliauskas, Paquet, SS, Teaney PRL 122 (2019) no.12, 122302; PRC 99 (2019) no.3, 034910

Establishing missing link between initial state and finale state important to connect Heavy-Ion Physics to small-x Physics at EIC & Hadron Colliders

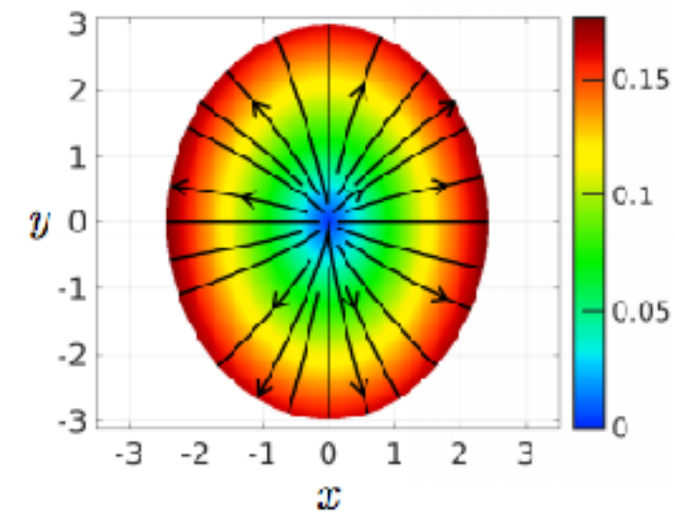
Small-x gluon (G)TMDs determine initial state energy density in p/A+A as well as e.g. (forward) di-jet production in p+p/A or e+p/A

Backup

Pre-flow ($T^{\tau i}$) & Viscous corrections (T^{ij})

Gradients in x_T induce off-diagonal components of $T^{\mu\nu}$

P. Chesler JHEP 1603 (2016) 146

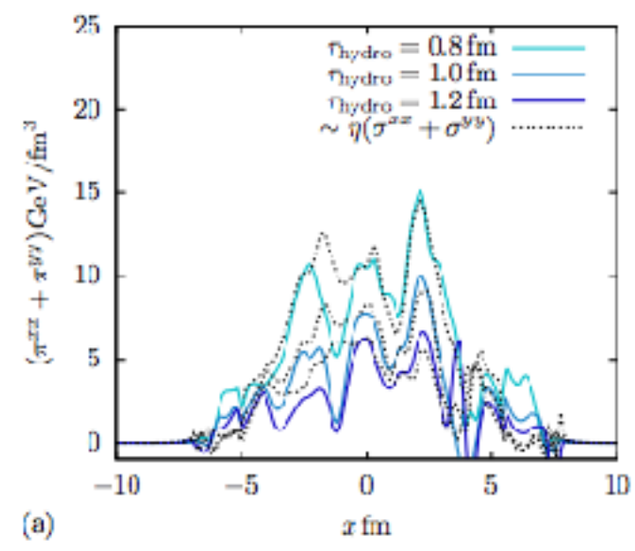


Universal pre-flow: early expansion of matter insensitive to microscopic details

Pratt, Vredevoogd PRC79 (2009) 044915
Kurkela, Mazeliauskas, Paquet, SS, Teaney PRC 99 (2019) no.3, 034910

$$\frac{T^{\tau i}(\tau,\mathbf{x})}{T^{\tau\tau}(\tau,\mathbf{x})} \approx -\frac{(\tau-\tau_0)}{2}\frac{\partial^i T^{\tau\tau}(\tau_0,\mathbf{x})}{T^{\tau\tau}(\tau,\mathbf{x})}$$

not indicative of onset of hydrodynamic flow



Viscous corrections: long wavelength components satisfy hydrodynamic constitutive equations approximately at $\tau=\tau_{Hydro}$

$$\pi^{\mu\nu}(\tau_{\rm Hydro},\mathbf{x}) \approx \pi^{\mu\nu}_{\rm NS}$$

Pre-flow package KoMPoST provide unified description of pre-equilibrium evolution of $T^{\mu\nu}$ based on linear response to gradients

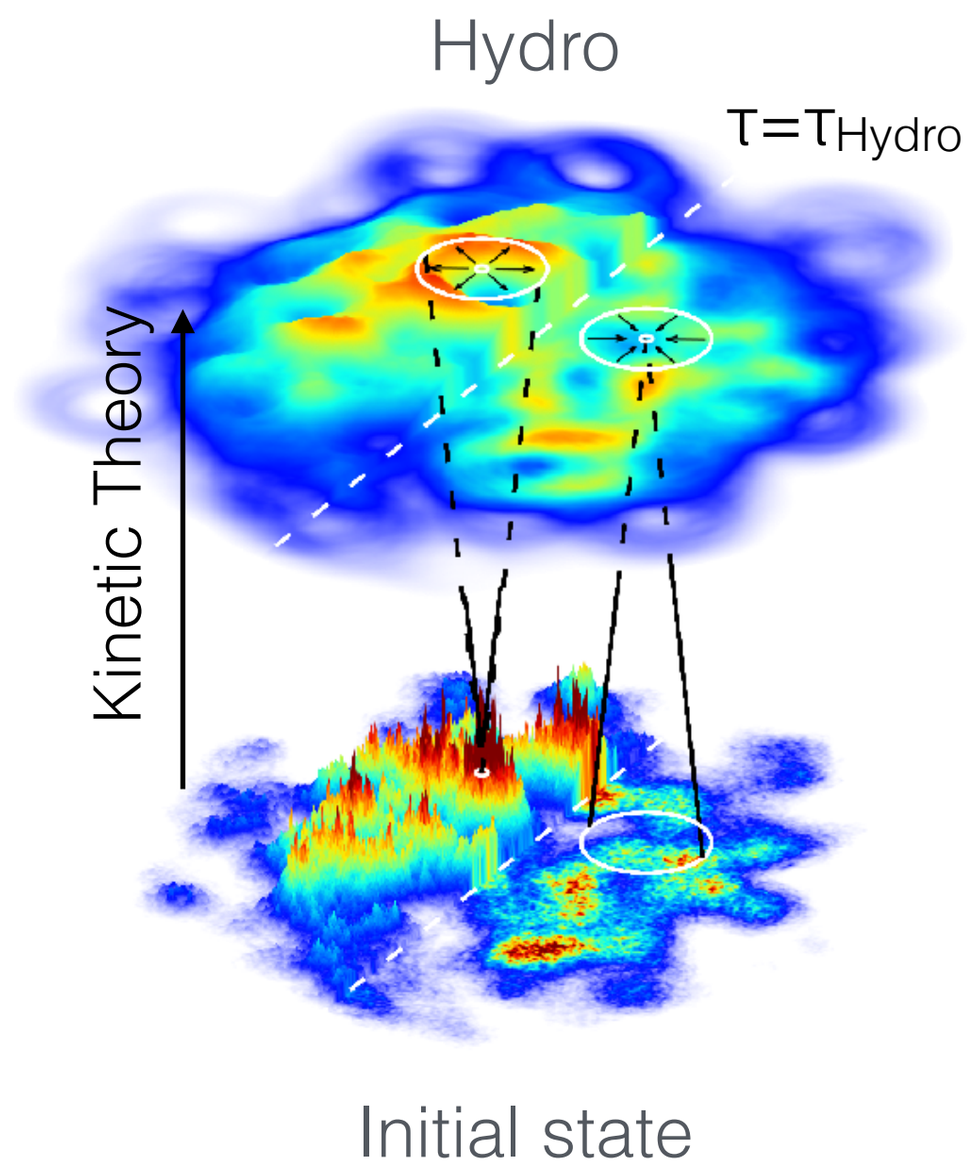
KoMPoST

Macroscopic description of pre-equilibrium dynamics

Exploit memory loss to use macroscopic degrees of freedom $T^{\mu\nu}$ for description of pre-equilibrium dynamics

Separation of scales between evolution time τ_{Hydro} and typical size scale of gradients R_A

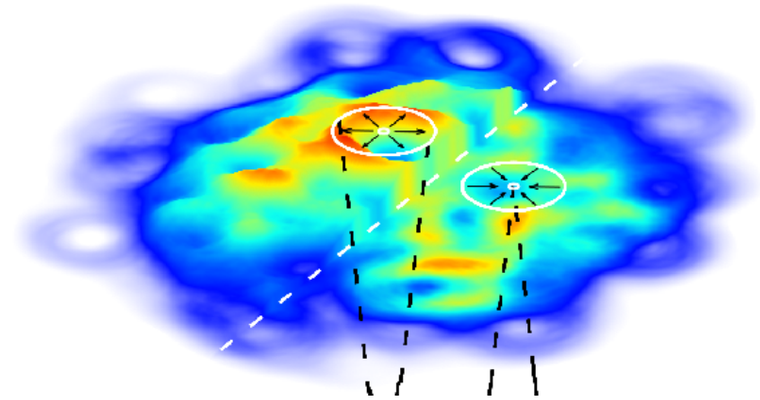
Decomposing $T^{\mu\nu}(x)$ into a local average $T^{\mu\nu}_{BG}(x)$ and fluctuations $\delta T^{\mu\nu}(x)$,are small on scales $c(\tau_{Hydro}-\tau_0)$, they can be treated in linear response theory



Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171
Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961

Non-equilibrium linear response

Energy-momentum tensor at τ_{Hydro} can be reconstructed directly from initial conditions

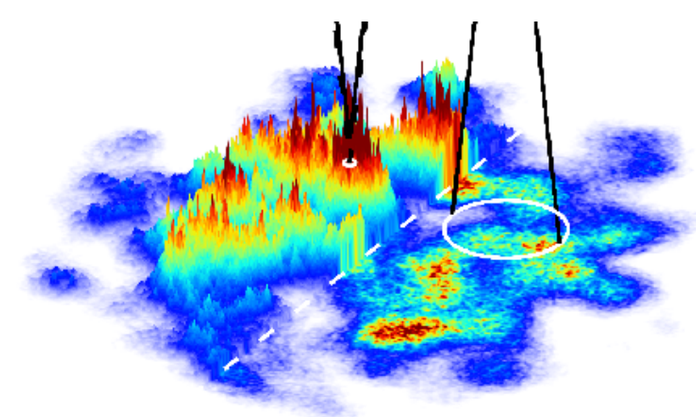


$$T^{\mu\nu}(\tau, x) = T^{\mu\nu}_{BG}(\tau) + \int_{\odot} G^{\mu\nu}_{\alpha\beta}(\tau, \tau_0, x, x_0)\delta T^{\alpha\beta}(\tau_0, x_0)$$

non-equilibrium evolution of (local) average background

non-equilibrium Greens function of energy-momentum tensor

Instead of event-by-event Monte-Carlo, effective kinetic theory simulations performed only once to compute evolution of background $T^{\mu\nu}_{BG}$ and Greens functions $G^{\mu\nu}_{\alpha\beta}$



Kurkela, Mazeliauskas, Paquet, SS, Teaney arXiv:1805.01604; arXiv:1805.00961