Non-equilibrium physics WS 18/19 – Exercise Sheet 12:

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1 Discussion:

i) What is a stochastic differential equation, and what are strategies to solve them?

ii) What is a fluctuation dissipation relation?

2 In-class problems:

2.1 Brownian motion

Consider the Brownian motion of a particle in one dimension, described by the stochastic differential equation

$$M\frac{dv}{dt} = -M\gamma v(t) + F_L(t) \ , \tag{1}$$

where $F_L(t)$ is the stochastic Langevin Force with $\langle F_L(t)\rangle = 0$ and $\langle F_L(t)F_L(t')\rangle = \kappa(t-t')$.

i) Derive a general expression for the un-equal time correlation function $\langle F_L(t)v(t')\rangle$ of Langevin force F_L and the velocity v in terms of κ.

ii) Calculate un-equal time correlation function $\langle F_L(t)v(t')\rangle$ explicitly for $\kappa(\Delta t) = 2D_v M^2\delta(\Delta t)$ and discuss the features of your result.

3 Homework problems:

3.1 Brownian motion

Consider the Brownian motion of a particle in one dimension, described by the stochastic differential equation

$$M\frac{dv}{dt} = -M\gamma v(t) + F_L(t) \ , \tag{2}$$

where $F_L(t)$ is the stochastic Langevin Force with $\langle F_L(t)\rangle = 0$ and $\langle F_L(t)F_L(t')\rangle = \kappa(t-t')$.

i) Derive a general expression for the un-equal time correlation function $\langle v(t)v(t')\rangle$ of the velocity v in terms of κ.

ii) Calculate un-equal time correlation function $\langle v(t)v(t')\rangle$ explicitly for $\kappa(\Delta t) = 2D_v M^2\delta(\Delta t)$ and discuss the features of your result.

3.2 Brownian motion in the viscous limit

In the first theories of Brownian motion proposed by A. Einstein and M. Smoluchowski, the process was described in a simpler way according to

$$M\gamma \frac{dx}{dt} = F_L(t) \tag{3}$$

where $F_L(t)$ is a stochastic force with vanishing expectation value $\langle F_L(t)\rangle = 0$ and auto-correlation function $\langle F_L(t)F_L(t')\rangle = 2DM^2\gamma^2\delta(t-t')$.

i) Show that the model in Eq. (3) can be obtained from the equation for Brownian motion in Eq. (2) in the combined limit $M \to 0$ $\gamma \to \infty$ with $M\gamma$ held constant.

ii) Calculate the average displacement $\langle x(t) - x(t_0)\rangle$ and its variance $\langle [x(t) - x(t_0)]^2\rangle$ for $t > t_0$ and comment on the similarities and differences between the results obtained for the process in Eq. (2) (discussed in the lecture) and the simplified model process in Eq. (3).