Non-equilibrium physics WS 18/19 – Exercise Sheet 14:

Universität Bielefeld
Instructors: Jun.-Prof. Dr. S. Schlichting, D. Schröder

1 Discussion:

i) What is the physical meaning of a generalized susceptibility? What is the Kubo formula?

2 In-class problems:

2.1 Spectral function

We consider the spectral function $\tilde{\rho}_{BA}(\omega) = \frac{1}{\hbar} \int_{-\infty}^{\infty} dt \left\langle [B_I(t), A_I(0)] \right\rangle_{\text{eq}} e^{i\omega t}$ which is related to the generalized susceptibility $\tilde{\chi}_{BA}(\omega)$ through the relation

$$\tilde{\chi}(\omega) = \frac{1}{2\pi} \lim_{\epsilon \to 0^+} \int_{-\infty}^{\infty} d\omega' \frac{\rho_{BA}(\omega')}{\omega' - \omega - i\epsilon} \tag{1}$$

i) Show that the spectral function satisfies the relation $\tilde{\rho}_{BA}(\omega) = -\tilde{\rho}_{AB}(-\omega)$ in frequency space

ii) If you assume that the spectral function is real, i.e. $\tilde{\rho}_{BA}(\omega) = \left(\tilde{\rho}_{BA}(\omega)\right)^*$ what can you say about the real and imaginary parts of the generalized susceptibility $\chi_{BA}(\omega)$ and $\chi_{AB}(\omega)$. $\left(\text{Hint: } \lim_{\epsilon \to 0^+} \frac{1}{x+i\epsilon} = pv\frac{1}{x} - i\pi\delta(x)\right)$