Effects of reheating on the expansion history

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Overview

Basic Problem

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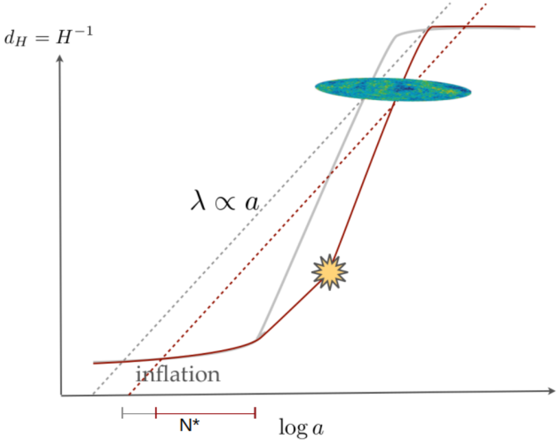

- signals from inflation can be observed in the CMB today
- uncertainties in expansion history during reheating create uncertainties in origin of the signals

Signals from Inflation

Signals from Inflation: Introducing perturbations

introduce perturbations of the metric and energy momentum tensor (scalar, vector, tensor) with wavelength $\lambda \Leftrightarrow$ wavenumber k.

$$g^{\mu\nu}(x^\alpha) = \bar{g}^{\mu\nu}(t) + \delta g^{\mu\nu}(x^\alpha) \ , \quad T^{\mu\nu}(x^\alpha) = \bar{T}^{\mu\nu}(t) + \delta T^{\mu\nu}(x^\alpha) \ \ ,$$
$$\phi(x^\alpha) = \bar{\phi}(t) + \delta\phi(x^\alpha)$$

- these are perturbations of the inflaton field (which the universe mainly consists of)
- next, calculate evolution using GR and inflaton dynamics

Signals from Inflation: Introducing perturbations

- vector perturbations redshifted away
- scalar & tensorial perturbations will freeze when they leave the Hubble horizon ($k \lesssim aH$) as gaussian field fluctuations with power spectra

$$\langle 0|\mathcal{R}(\tau,\mathbf{x})\mathcal{R}(\tau,\mathbf{x})|0\rangle =: \int \mathrm{d}\ln k\, \Delta^2_{\mathcal{R}}(\tau,k)$$

$$\langle 0|4h^+(\tau,\mathbf{x})h^+(\tau,\mathbf{x}) + 4h^\times(\tau,\mathbf{x})h^\times(\tau,\mathbf{x})|0\rangle =: \int \mathrm{d}\ln k\, \Delta^2_t(\tau,k)$$

$$\text{where} \quad \Delta^2_{\mathcal{R}}(k) \approx \left.\frac{H^2}{8\pi^2 m_{Pl}^2 \epsilon_H}\right|_{k=aH} \quad , \quad \Delta^2_t(k) \approx \left.\frac{2H^2}{\pi^2 m_{Pl}^2}\right|_{k=aH}$$

Reminder: slow roll parameter

$$\epsilon_H = \dot{H}/H^2$$

- scale invariant during inflation, k-dependence only through exit of Hubble horizon

Signals from Inflation: Key observables

k-dependence of $\Delta^2_{\mathcal{R}}$ and Δ^2_t can be approximated by simple power laws; can be written as

$$\Delta^2_{\mathcal{R}} \approx A_s \left(\frac{k}{k_*}\right)^{n_s-1} \quad , \quad \Delta^2_t \approx A_t \left(\frac{k}{k_*}\right)^{n_t}$$

- k_*: arbitrary fixed reference scale, called "pivot scale"; starred variables refer to the respective value when $k_* = a_* H_*$
- A_t is typically described by the tensor-to-scalar ratio $r = A_t / A_s$

Signals from Inflation: Key observables

- slow roll inflation predicts: inflation observables depend on $V(\phi_*)$ and its derivatives:

$$n_s - 1 \approx -6\epsilon_V + 2\eta_V \quad , \quad r = -8n_t \approx 16\epsilon_V \quad , \quad A_s \approx \frac{V}{24\pi^2 m_{Pl}^4 \epsilon_V}$$

Reminder: slow roll parameters

$$\epsilon_V = \frac{m_{Pl}^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2 \quad , \quad \eta_V = m_{Pl}^2 \frac{\partial_\phi^2 V}{V}$$

- $r = -8n_t$: "consistency relation for slow roll inflation"
- these parameters can be measured from CMB observations: $A_s = 2.2 \times 10^{-9}$, $n_s = 0.968 \pm 0.006$, $r < 0.11$
- **but:** comparison with inflation model predictions require knowledge of ϕ_* $\Rightarrow$ require knowledge of inflation history

Expansion history

Expansion history: How does N_* vary?

main object of interest: number of e-folds between a_* and a_{end} can be calculated assuming conservation of entropy after thermalization[1,2]:

$$N_* = 66.89 - \ln\frac{k_*}{a_0 H_0} + \frac{1}{4}\ln\frac{V_*^2}{m_{Pl}^4 \rho_{end}} + \frac{1}{12}\ln\left[g_{th}^{-1}\left(\frac{\rho_{th}}{\rho_{end}}\right)^{\frac{1-3\bar{w}}{1+\bar{w}}}\right]$$

- to get this formula, need to match pivot scale today to the time of horizon exit: $\frac{k*}{a_0 H_0} = \frac{a_* H_*}{a_0 H_0}$, take the log and split $\frac{a_* H_*}{a_0 H_0}$ into multiple fractions for the different stages of expansion
- everything that is known (matter domination, radiation domination) can be calculated and accumulated into the constant 66.89

[1] Planck Collaboration (2014), AA, 571, A22, arXiv:1303.5082

[2] K. D. Lozanov and M. A. Amin, Phys. Rev. D97 no. 2, (2018) 023533, arXiv:1710.06851

Expansion history: How does N_* vary?

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parameters that N_* depends on:

1. k_* is arbitrary, needs to be fixed
2. V_* depends on the model of inflation
3. thermal properties of reheating:
   - average equation of state during reheating $\bar{w} := (t_{th} - t_{end})^{-1} \int_{t_{end}}^{t_{th}} \mathrm{d}t\, w(t)$
   - effective relativistic degrees of freedom at thermalization g_{th}
   - energy density at thermalization ρ_{th}

Expansion history: Parameter dependency

$$N_* = N_* \left(V_*, g_{th}^{-1} \left(\frac{\rho_{th}}{\rho_{end}} \right)^{\frac{1-3\bar{w}}{1+\bar{w}}} \right)$$

Step 1: Let's take a closer look at V. Depending on the model of inflation, the potential will have some free parameters $\{q_i\}$, so we have $V_* = V_*(\{q_i\}, \phi_*)$.

Example: $V = \frac{m^2}{2}\phi^2$

$$V_* = V(\phi_*) = \frac{m^2}{2}\phi_*^2 = V_*(m, \phi_*)$$

Expansion history: Parameter dependency

$$N_* = N_* \left(V_*, g_{th}^{-1} \left(\frac{\rho_{th}}{\rho_{end}} \right)^{\frac{1-3\bar{w}}{1+\bar{w}}} \right) \ , \quad V_* = V_*(\{q_i\}, \phi_*)$$

Step 2: In the end, ϕ is in a potential minimum, $\phi_{end} = \phi_{end}(\{q_i\})$ and since $N_* \approx \left| \int_{\phi_*}^{\phi_{end}} \frac{\mathrm{d}\phi}{m_{Pl}} \frac{1}{\sqrt{2\epsilon_V}} \right|$, we have $\phi_* = \phi_*(N_*, \{q_i\})$

Example: $V = \frac{m^2}{2}\phi^2$

$$N_* = \left| \int_{\phi_*}^{\phi_{end}} \frac{\mathrm{d}\phi}{m_{Pl}} \sqrt{\frac{2}{m_{Pl}^2} \left(\frac{\partial_\phi V}{V} \right)^2} \right| = \left| \int_{\phi_*}^{\phi_{end}} \frac{\mathrm{d}\phi}{m_{Pl}} \sqrt{\frac{\phi^2}{2m_{Pl}^2}} \right| = \frac{\left| \phi_{end}^2 - \phi_*^2 \right|}{\sqrt{8} m_{Pl}^2}$$

$$\Rightarrow \phi_* = \sqrt{\sqrt{8} m_{Pl}^2 N_* - \phi_{end}^2(m)} = \phi_*(m, N_*)$$

Expansion history: Parameter dependency

$$N_* = N_*\left(V_*, g_{th}^{-1}\left(\frac{\rho_{th}}{\rho_{end}}\right)^{\frac{1-3\bar{w}}{1+\bar{w}}}\right)\ , \quad V_* = V_*(\{q_i\}, \phi_*)\ , \quad \phi_* = \phi_*(\{q_i\}, N_*)$$

Step 3: A_s is a function of ϕ_*, so its measurement can be used to fix one of the parameters, $q_1 = q_1(\{q_i\} \setminus \{q_1\}, N_*, A_s)$

Example: $V = \frac{m^2}{2}\phi^2$

$$A_s = \frac{V^3}{12\pi^2 m_{Pl}^6 (\partial_\phi V)^2} = \frac{m^2\phi_*^4}{48\pi^2 m_{Pl}^6}$$

$$\Rightarrow m^2\phi_*^4(m, N_*) = 48\pi^2 m_{Pl}^6 A_s \Rightarrow \mathrm{m}^2 = \frac{6\pi^2 m_{Pl}^2 A_s}{N_*^2} = m^2(N_*, A_s)$$

Next, we can put all of this together!

Expansion history: Parameter dependency

In total:

$$N_* = N_*\left(\{q_i\} \setminus \{q_1\}, A_s, g_{th}^{-1}\left(\frac{\rho_{th}}{\rho_{end}}\right)^{\frac{1-3\bar{w}}{1+\bar{w}}}\right)$$

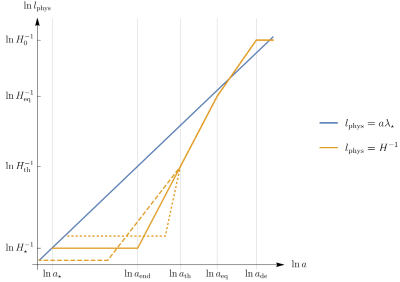
$$\text{and} \quad V_* = V_*(\{q_i\} \setminus \{q_1\}, A_s, N_*)$$

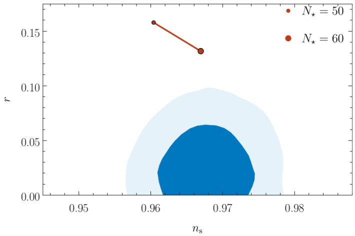
- If there is only one parameter, it will be fixed by the measurement of A_s and we have a one-to-one correspondance between V_* and N_*!
- In general, for a given model of inflation with l free parameters, there will be an l-dimensional space of solutions!
- For each solution, the other inflation observables n_s and r can be computed and compared with observations.

Model predictions

Model predictions

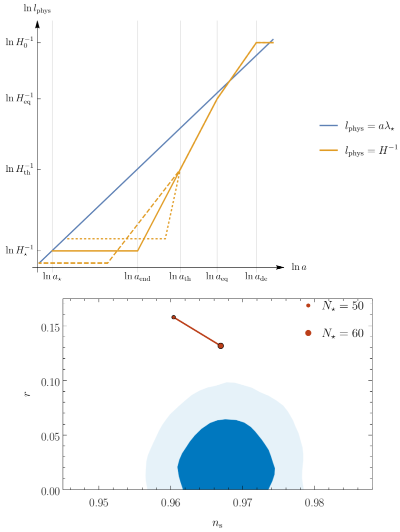
$$V = \frac{m^2}{2}\phi^2$$



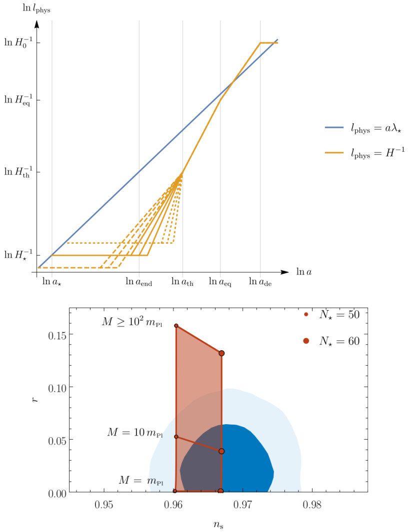


Model predictions

$$V = \frac{m^2}{2}\phi^2$$

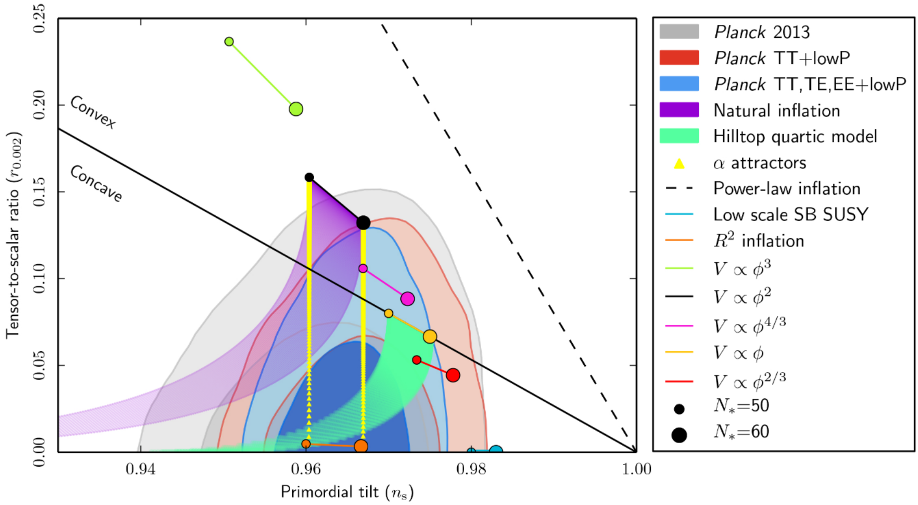


$$V = \Lambda^4 \tanh^2 \frac{\phi}{M}$$



Model Predictions

"Planck 2015 results. XX. Constraints on inflation", arXiv: 1502.02114



Summary

- Signals from inflation can be observed in the CMB today. Describing their origin requires modelling inflation and reheating.
- Uncertainties of thermal properties of reheating translate into one "dimension" of uncertainty in the computation of inflation observables
- If one of the inflation observables is used to fix one of the inflaton potential parameters, each additional parameter beyond this first one also adds one "dimension" of uncertainty
- Model predictions and measurements of the other inflation observables can be plotted together to see how well they fit and exclude certain models.