SCENARIOS OF REHEATING AND INFLATON DECAY

THERMALIZATION OF THE EARLY UNIVERSE

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June 26, 2020

Universität Bielefeld

Based on :

- K.Harigaya, K.Mukaida, "Thermalization after/during Reheating", arXiv:1312.3097
- K.Mukaida, M. Yamada, "Thermalization Process after Inflation and Effective Potential of Scalar Field", arXiv:1506.07661

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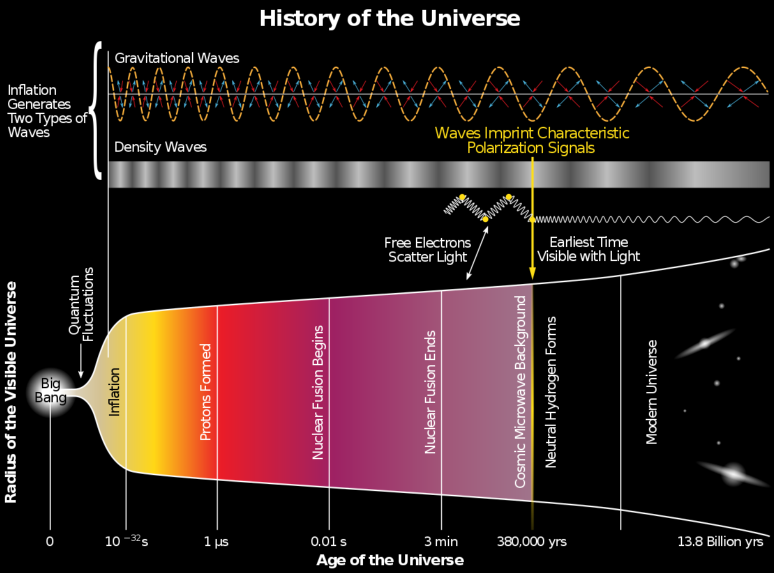
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10 $^{-32}$s
1 μs
0.01 s
3 min
380,000 yrs
13.8 Billion yrs
Age of the Universe

BIG PICTURE

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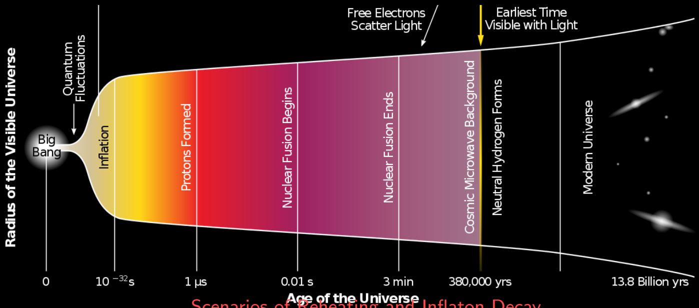

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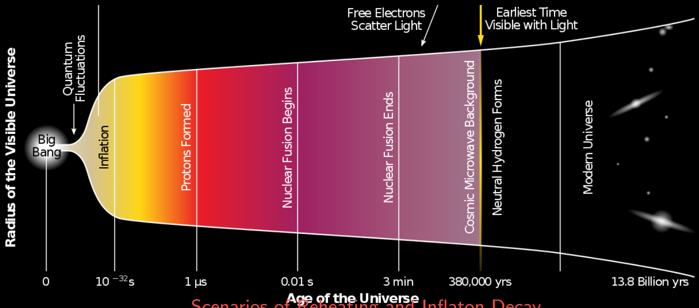

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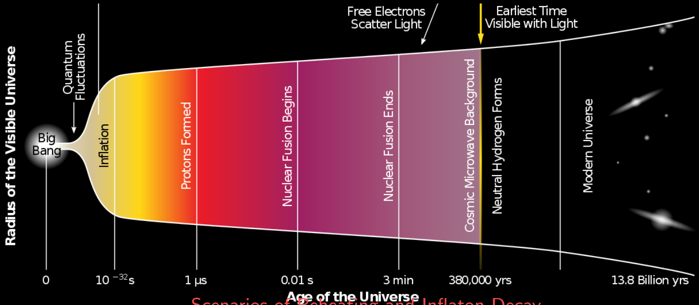

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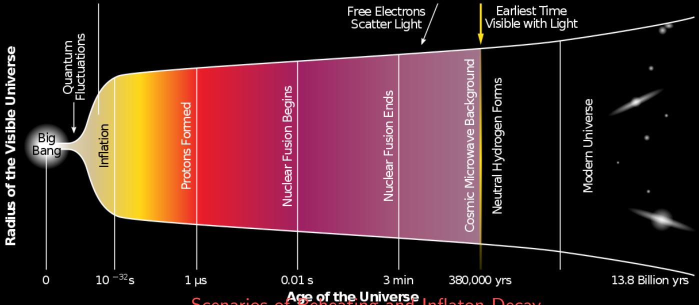

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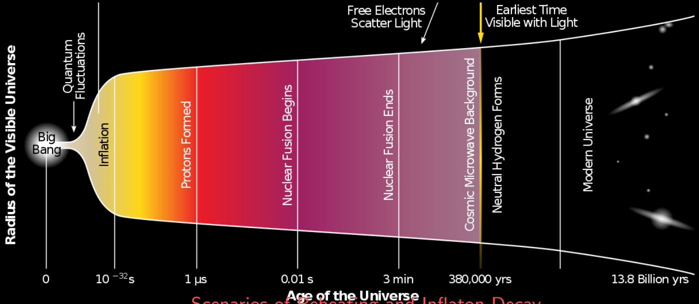

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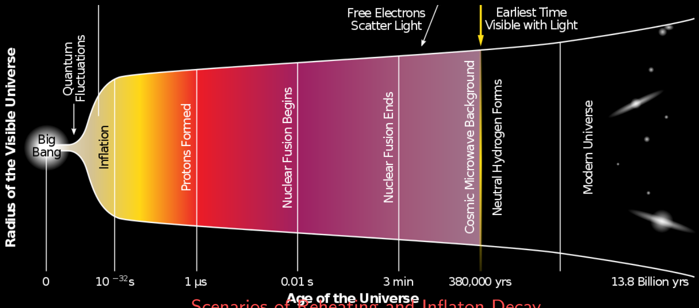

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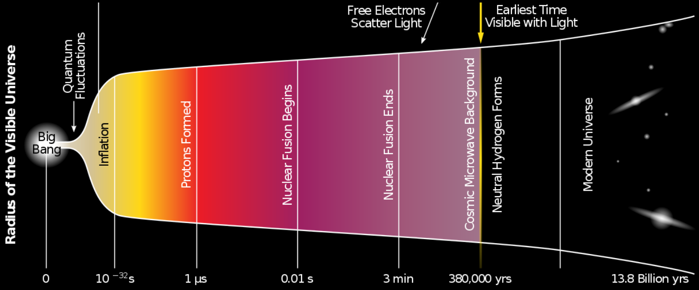

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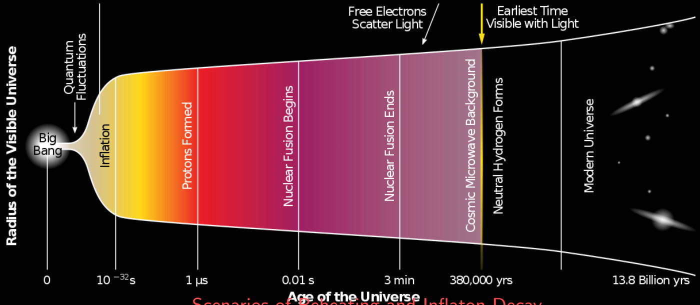

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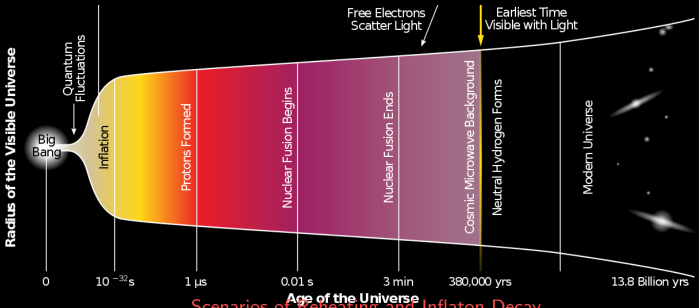

REHEATING

REHEATING: EQUATIONS OF MOTION [1]

Scalar field $\phi(t)$ loses energy through Universe expansion (Hubble friction) $\Rightarrow$ decays into lighter particles.

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$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_\phi + \left(3H + \Gamma_\phi\right)\rho_\phi = 0, \qquad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_r + 4H\rho_r - \Gamma_\phi\rho_\phi = 0, \qquad (2)$$

$$3H^2 M_{\mathrm{pl}}^2 = \rho_\phi + \rho_r, \qquad (3)$$

where

- ρ_ϕ : Energy density of scalar Field
- ρ_r : Energy density of decay products
- Γ_ϕ : Decay rate of scallar field

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REHEATING: SOLUTION

Energy density dominated by $\phi(t) \Rightarrow H = \frac{\sqrt{\rho_\phi}}{(\sqrt{3}M_{\rm pl})}$ and decay width negligible to Hubble parameter.

$$\rho_\phi \simeq \frac{4}{3}\frac{M_{\rm pl}^2}{t^2}, \quad \rho_r \simeq \frac{4}{5}\Gamma_\phi M_{\rm pl}^2 \frac{1}{t} \sim \rho_\phi \Gamma_\phi t, \quad H = \frac{2}{3t}. \tag{4}$$

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Energy density dominated by particles with momentum $p \sim m_\phi$

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Lower scales are populated by redshift

$$f(p) \sim \begin{cases} \left(\frac{\Gamma_\phi M_{\rm pl}^2}{m_\phi^3}\right)(m_\phi t)^{-1}\left(\frac{m_\phi}{p}\right)^{3/2} & \text{for} \quad (t/t_i)^{-2/3} m_\phi \lesssim p \lesssim m_\phi \\ 0 & \text{for} \quad \text{otherwise} \end{cases}, \tag{6}$$

t_i: time at which oscillation of ϕ dominates energy of the universe.

END OF REHEATING

When $\epsilon_r \sim \epsilon_\phi$ at $t^{-1} \sim H \sim \Gamma_\phi \Rightarrow$ This is the end of reheating.

Reheating Temperature:

$$3\Gamma_\phi^2 M_{\mathrm{pl}}^2 \equiv \frac{\pi^2}{30} g_* T_{\mathrm{rh}}^4, \tag{7}$$

If thermalization is fast enough reheating temperature T_{rh} should be the temperature of the universe at that point.

THERMALIZATION

AFTER REHEATING

After reheating we write:

- The number density of the decay products

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Consider two type of interactions:

order one Yukawa coupling	dimension 5 Planck-suppressed
$\Gamma_\phi \simeq m_\phi$	$\Gamma_\phi \simeq m_\phi^3/M_{\rm pl}^2$
$n_{\rm th}/n_i \simeq \sqrt{m_\phi/M_{\rm pl}} \ll 1$	$n_{\rm th}/n_i \simeq \sqrt{M_{\rm pl}/m_\phi} \gg 1$
Over-Occupied	Under-Occupied

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$$\sigma_i \simeq k \frac{1}{p_i^2} \simeq k \frac{1}{m_\phi^2}, \qquad k \simeq \frac{g^6}{128\pi^3}, \tag{8}$$

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This ratio is larger than one if

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Thermalization when the Universe becomes k^{-1} times large. The thermalized temperature $T_{\rm th} \simeq k T_{\rm rh} < T_{\rm rh}$ even if the QCD interaction is involved.

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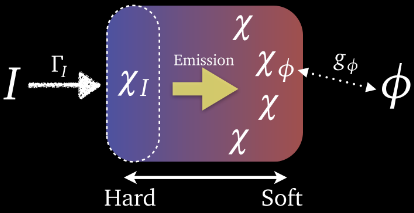
  - as we have learned from guage thermalization, Collinear radiation plays an important role
  - The inelastic small angle interaction rate are enhanced even the energy loss is suppressed

SETUP

Now I will talk about the second paper [2], where they consider a thermalization following an effective kinetic theory (similar to what A. Klaus presented for QCD).



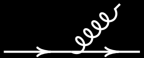
- I inflaton creates hard primaries χ_I with typical momentum m_I
- χ_I emits light SM fields χ (including χ_I) and massive SM fields χ_ϕ (through SM interactions)
- ϕ scalar field $\neq$ inflaton
- χ_ϕ aquire their mass from the coupling to ϕ condensate $m = |g_\phi \phi|$
- $\tilde{t} = m_I t$ cosmic time normalized by the inflaton mass.

When potential force overcomes expansion $H \sim \sqrt{|\partial V_{\text{eff}}/\partial \phi^2|}$, ϕ relaxes to its "equilibrium value" by dissipating its energy into the background plasma.

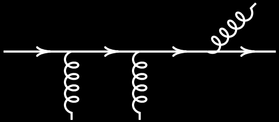
[2] arXiv:1506.07661

Let us take an effective kinetic theory:

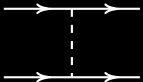
$$\left[\partial_t - H\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{k}}\right] f_a(t,k) = -\Big((\mathcal{C}_{\text{decay}}) + \mathcal{C}_{\text{split}} + \mathcal{C}_{2\longleftrightarrow 2}\Big)[f_a], , \tag{11}$$



Decay.



Inelastic Collinear radiation: LPM.



Elastic processes → broadening $\hat{q}_{\rm el}$.

■ $2 \longleftrightarrow 2$ processes: The elastic processes are characterized by the momentum broadening constant

$$\hat{q}_{\mathrm{el}} \sim \int d^2 q_\perp \frac{d\Gamma_{\mathrm{el}}}{dq_\perp^2} q_\perp^2 \sim \alpha^2 \int_{p'} f_\bullet(p')[1 \pm f_\bullet(p')], \tag{12}$$

$$\frac{d\Gamma_{\mathrm{el}}}{dq_\perp^2} \sim \frac{\alpha^2}{q_\perp^2(q_\perp^2 + m_s^2)} \int_{\mathbf{p}'} f_\bullet(p')[1 \pm f_\bullet(p')]. \tag{13}$$

Note that the screening mass for χ-particles is given by $m_s^2 \sim \alpha T_*^2$.

THERMALIZATION: LPM SPLITTINGS

■ Collinear radiation: Subsequent splittings are suppressed due to interference effects.

$$\Gamma_{\text{split}}(k) \sim \alpha \; Min\left[\Gamma_{\rm el}, \; \frac{1}{t_{\text{form}}}\right], \quad t_{\text{form}} \equiv \sqrt{\frac{k}{\hat{q}_{\rm el}}}. \tag{14}$$

t_{form} : formation time which it takes to resolve the radiation from the parent.
k : radiation momentum.
The maximum radiation momentum due to LPM suppression

$$k_{\text{form}} = \hat{q}_{\text{el}} t^2. \tag{15}$$

THERMALIZATION: DECAY

- χ_ϕ decay :

$$\mathcal{C}_{\rm dec}[f_{\chi_\phi}] \sim \int_{\mathbf{k}',\mathbf{p}'} \frac{\left|\mathcal{M}_{\rm dec}(K;K',P')\right|^2}{2E_k 2E_{k'} 2E_{p'}} (2\pi)^4 \delta^{(4)}\left(K-K'-P'\right) \left[f_{\chi_\phi}(k)\left[1\pm f_\bullet(k')\right]\left[1\pm f_\bullet(p')\right] - (\text{inverse process})\right]. \quad (16)$$

- Depends on models of χ_ϕ-couplings to other particles.
- Have to specify interaction terms which can induce decay of χ_ϕ into light particles for a large mass $|g_\phi \phi| > m_{\rm screen}$
- Assume that the typical magnitude of this term is $\propto \epsilon^2 \alpha$ with ϵ being a small parameter,
- and $\epsilon^2 \lesssim \alpha$

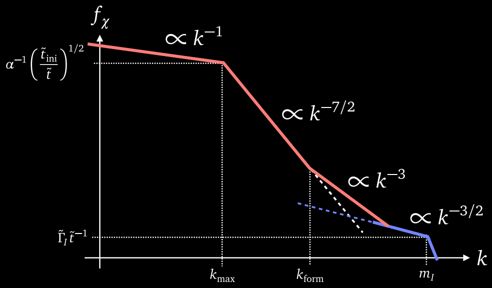
THERMALIZATION HISTORY

THERMALIZATION: INITIAL χ PRODUCTION

- $\tilde{t}_{\rm ini} < \tilde{t} < \tilde{t}_{\rm soft}$: χ production

To ensure causality wavelength scale ($k_{\rm max}^{-1}$) should be smaller than the Hubble horizon $H^{-1} \sim t$. $\Rightarrow k_{\rm max} > H$

$$\tilde{t} > \tilde{t}_{\rm ini} \equiv \alpha^{-1}\Gamma_I^{-1/2}. \tag{17}$$



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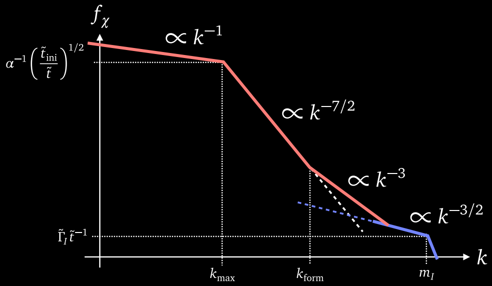

At first the hard primaries $p \simeq m_I$ scatter among themselves and emit soft particles through $\mathcal{C}_{\mathrm{split}}$.

$$f_{\mathrm{soft}}(t,k) \sim \Gamma_{\mathrm{split}}(k) n_{\mathrm{hard}} k^{-3} t, \tag{17}$$

$n_{\mathrm{soft}} \gg 1$ are over-pccupied $\Rightarrow$ inverse processes and elastic scattering processes have to be considered. As well as scatterings with hard primaries.

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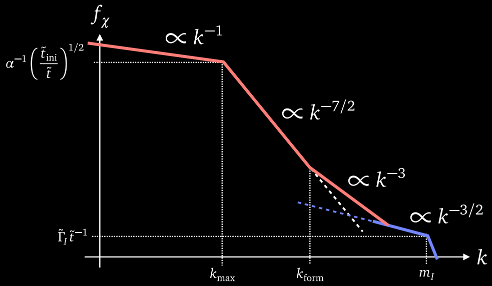

The soft particles thermalize to

$$f_{\rm soft}(t,k) \sim \frac{T_{\rm soft}}{k} \quad \text{for} \quad k < k_{\rm max}, \tag{17}$$

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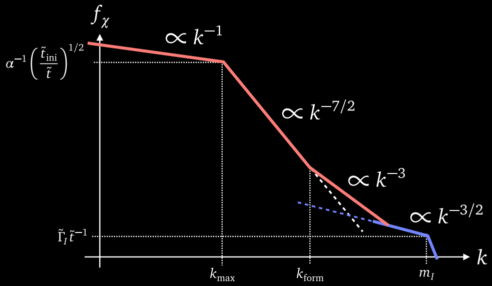


The soft particles thermalize to

$$f_{\rm soft}(t,k) \sim \frac{T_{\rm soft}}{k} \quad \text{for} \quad k < k_{\rm max}, \tag{17}$$

The effective temperature T_* is dominated by the soft sector

$$T_*^2 \sim \int_{\mathbf{k}} \frac{f_{\rm soft}}{k} \sim T_s k_{\rm max} \sim \alpha^{1/2}\Gamma_I^{3/4}\tilde{t}^{-1/2} m_I^2 \tag{18}$$

$$> \int_{\mathbf{k}} \frac{f_{\rm hard}}{k} \sim \Gamma_I \tilde{t}^{-1} m_I^2, \tag{19}$$

THERMALIZATION: INITIAL χ_ϕ PRODUCTION

$\tilde{t}_{\rm ini} < \tilde{t} < \tilde{t}_{\rm soft}$: χ_ϕ production

Let us consider that $|g_\phi \phi| > m_{\rm screen}$ (other case is the similar to χ).

- Splitting:

$$f_{\chi_\phi} \simeq \Gamma_{\rm split}(k) n_{\rm hard} k^{-3} t \to T^2_{*,\chi_\phi}|_{\rm hard} \simeq \Gamma_{\rm split}(M) n_{\rm hard} M^{-1} t, \quad (20)$$

where

$$M = \mathrm{Max}[|g_\phi \phi|, k_{\max,\chi_\phi}] \quad (21)$$

- $2 \longleftrightarrow 2$ processes:

Produced either from soft particles or from interactions between soft and hard particles

$$T^2_{*,\chi_\phi}\Big|_{\rm soft/int} \sim \frac{\alpha^2}{|g_\phi \phi|^3} t \int d\log k' \;\; n_{\rm soft}(k') n_{\rm soft/hard}\left(\frac{|g_\phi \phi|^2}{k'}\right), \quad (22)$$

THERMALIZATION: INITIAL χ_ϕ PRODUCTION

- $\tilde{t}_{\rm ini} < \tilde{t} < \tilde{t}_{\rm soft}$: χ_ϕ production
  - Decay:

The inverse decay can produce χ_ϕ particles

$$T^2_{*,\chi_\phi}\Big|_{\rm soft/int} \sim \frac{\epsilon^2\alpha}{|g_\phi\phi|^3}\, t \int d\log k'\, n_{\rm soft}(k')\, n_{\rm soft/hard}\left(\frac{|g_\phi\phi|^2}{k'}\right). \tag{20}$$

If $\epsilon^2 \lesssim \alpha$

$$\left\langle\Gamma_{\rm decay}\right\rangle t \sim \epsilon^2\alpha|g_\phi\phi|t \lesssim \left(\frac{|g_\phi\phi|}{k_{\rm max}}\right)\left(\frac{\tilde{t}}{\tilde{t}_{\rm soft}}\right), \tag{21}$$

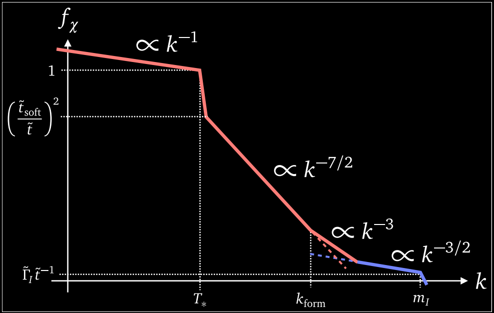
The decay is insignificant for $|g_\phi\phi| < k_{\rm max}$ at $\tilde{t} \lesssim \tilde{t}_{\rm soft}$.

THERMALIZATION:

- $\tilde{t}_{\rm soft} < \tilde{t} < \tilde{t}_{\rm max}$: χ production

$\tilde{t}_{\rm soft}$ is the time the soft sector is completely thermalized and can be described by $T_{\rm soft}$

$$\tilde{t} \sim \tilde{t}_{\rm soft} \equiv \alpha^{-3} \Gamma_I^{-1/2} \tag{22}$$



The thermal soft sector dominates the number density. The left hard primaries will lose their energy to the thermal bath

$$\hat{q}_{\rm el} \sim \alpha^2 \int_{p'} f_{\rm soft}(p')[1 \pm f_{\rm soft}(p')] \sim \alpha^2 T_{\rm soft}^3, \tag{23}$$

Vacuum shower turns into "in-medium" splittings.

THERMALIZATION:

- $\tilde{t}_{\mathsf{soft}} < \tilde{t} < \tilde{t}_{\max}$: χ_ϕ production

If $|g_\phi \phi| < T_{\rm soft}$, then their temperature is the same as soft χ particles

$$f_{\chi_\phi}(t,k) \sim \frac{T_s}{k} \quad \text{for} \quad k < T_s, \tag{24}$$

$$T^2_{*,\chi_\phi}\Big|_{\rm indir} \sim T_s^2, \tag{25}$$

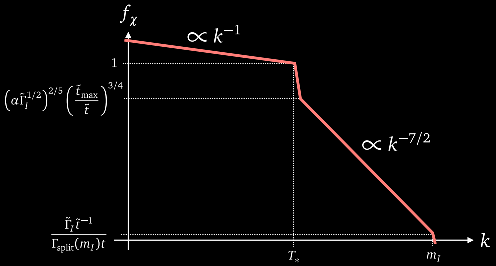
Because "hard" interactions among the soft sector with the momentum exchange of $T_{\rm soft}$ is much faster than the cosmic expansion.
χ_ϕ participate in the thermal plasma in this case.

THERMALIZATION:

- $\tilde{t}_{\max} < \tilde{t} < \tilde{t}_{\rm RH}$: χ production

When $\Gamma_{\rm split}(k \simeq m_\phi) \sim H$ i.e. when $k_{\rm split}$ becomes comparable to m_I, hard primaries lose their energy completely at

$$\tilde{t} \sim \tilde{t}_{\max} \equiv \alpha^{-16/5} \Gamma_I^{-3/5}. \quad (26)$$



The distribution of smaller wavenumber modes thermal and larger modes are determined by the LPM effect.

- $\tilde{t}_{\mathrm{max}} < \tilde{t} < \tilde{t}_{\mathrm{RH}}$: χ_ϕ production

Same as before, but here we have to consider the decrease of the number density of hard primaries by splittings for $t > t_{\max}$, We also take direct production from inflaton decay

$$f_{\chi_\phi}(t, m_I)\Big|_{\mathrm{dir}} \sim \tilde{\Gamma}_I \tilde{t}^{-1}\,\mathrm{Min}\left[\frac{1}{\Gamma_{\mathrm{split}}(m_I)t}, \frac{1}{\Gamma_{\mathrm{decay}}\,t}\right]. \quad (27)$$

END OF REHEATING

- $\tilde{t}_{\rm RH} < \tilde{t}$

$\Gamma_I \sim H(t) \Rightarrow$ the Universe is dominated by radiation and reheating is completed

$$\tilde{t} \sim \tilde{t}_{\rm RH} \equiv \Gamma_I^{-1} \frac{M_{\rm pl}^2}{m_I^2}. \quad (28)$$

The reheating temperature

$$T_{\rm RH} \sim \sqrt{\Gamma_I M_{\rm pl}}. \quad (29)$$

The Universe temperature will decreases with time

$$T_* \sim T_{\rm RH} \left[\frac{\tilde{t}_{\rm RH}}{\tilde{t}} \right]^{1/2}, \quad (30)$$

because $H^2 \sim T_*^4 / M_{\rm pl}^2$.

SUMMARY

SUMMARY

$$\frac{T_*}{m_\phi} \sim \begin{cases} \alpha^{1/2}\tilde{\Gamma}_I^{1/2}\left[\frac{\tilde{t}}{\tilde{t}\mathrm{ini}}\right]^{-1/4} & \text{for} \quad \tilde{t}\mathrm{ini} \lesssim \tilde{t} \lesssim \tilde{t}\mathrm{soft} \\ \alpha\tilde{\Gamma}_I^{1/2}\left[\frac{\tilde{t}}{\tilde{t}\mathrm{soft}}\right] & \text{for} \quad \tilde{t}\mathrm{soft} \lesssim \tilde{t} \lesssim \tilde{t}\mathrm{max} \\ \alpha^{4/5}\tilde{\Gamma}_I^{2/5}\left[\frac{\tilde{t}}{\tilde{t}\mathrm{max}}\right]^{-1/4} & \text{for} \quad \tilde{t}\mathrm{max} \lesssim \tilde{t} \lesssim \tilde{t}\mathrm{RH} \end{cases}$$

$$\tilde{t}\mathrm{ini} \equiv \alpha^{-1}\tilde{\Gamma}_I^{-1/2},$$
$$\tilde{t}\mathrm{soft} \equiv \alpha^{-3}\tilde{\Gamma}_I^{-1/2},$$
$$\tilde{t}\mathrm{max} \equiv \alpha^{-16/5}\tilde{\Gamma}_I^{-3/5},$$
$$\tilde{t}\mathrm{RH} \equiv \tilde{\Gamma}_I^{-1}\frac{M_{\mathrm{pl}}^2}{m_I^2}.$$

Thermalization was assumed to take place faster than reheating, true in most cases for: $\alpha \gtrsim 4 \times 10^{-4}(m_I/10^{13}\mathrm{GEV})^{5/8}\tilde{\Gamma}_I^{1/8}$.

SUMMARY

Evolution of effective temperature of soft sector. $H_I = m_I$, $m_I = 10^{13}\mathrm{GeV}$ and $\alpha = 0.1$

Figure 1: $\tilde{\Gamma}_I = 1 \Rightarrow \tilde{\Gamma}_I > \alpha^3$

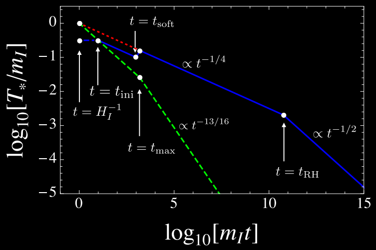
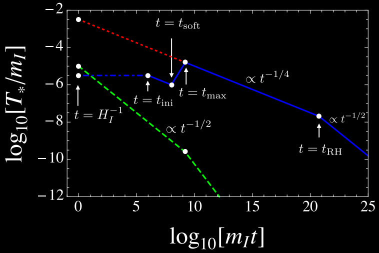


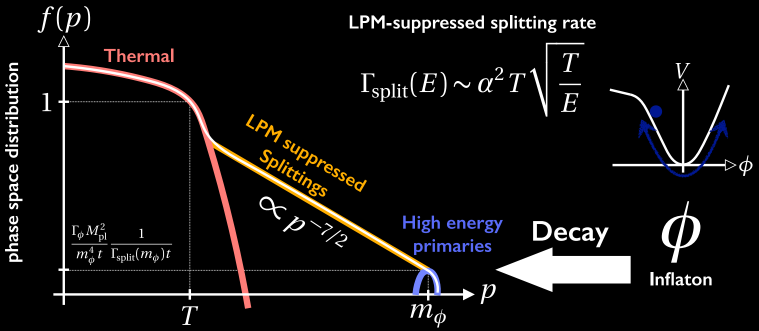
Figure 2: $\tilde{\Gamma}_I = 10^{-10} \Rightarrow \tilde{\Gamma}_I < \alpha^3$



- blue lines: effective temperature of soft sector
- red dotted lines: temperature derived in the literature by assuming the "instantaneous thermalization"
- green dashed lines: effective temperature of hard sector
- blue dot-dashed line: estimate of temperature for $\tilde{t} < \tilde{t}_{\rm init}$ using vacuum cascades

CONCLUSION

- For Planck-suppressed decay rates, hard primaries are under-occupied
- Collinear radiation is important for thermalization to be fast also for particle production



✦ Thermal population dominates both energy and number of radiation for $\Gamma_{split}(m_\phi) > H$.

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From talk by Kyohei Mukaida

BACKUP SLIDES

APPLICATION: PQ SYMMETRY

$$V(\phi) = m_T^2|\phi|^2 + \frac{\lambda^2}{2}(|\phi|^2 - \nu^2)^2, \quad (31)$$

with thermal mass ($m_T^2 \sim \alpha_\phi T_{*,\chi_\phi}^2$).
Symmetry breaking at T_{SSB} determined by $m_T(T_{\mathrm{SSB}}) \simeq \lambda v$.
For $m_T(T) < \lambda \nu$ we obtain upper bound for the reheating temperature

$$T_{\mathrm{RH}} \lesssim 2 \times 10^{10}\mathrm{GeV}\left[\frac{\alpha}{0.1}\right]^{-1}\left[\frac{\lambda v}{10^{12}\mathrm{GeV}}\right]\left[\frac{m_\phi}{10^{13}\mathrm{GeV}}\right]^{1/2}, \quad (32)$$

Using $\tilde{\Gamma}_I > \alpha^3$, $\alpha \sim \alpha_\phi$, and χ_ϕ (PQ-quarks) not produced directly from inflaton decay.

APPLICATION: PQ SYMMETRY

$$V(\phi) = m_T^2|\phi|^2 + \frac{\lambda^2}{2}(|\phi|^2 - v^2)^2, \tag{31}$$

$$T_{\rm RH} \lesssim 2 \times 10^{10}{\rm GeV}\left[\frac{\alpha}{0.1}\right]^{-1}\left[\frac{\lambda v}{10^{12}{\rm GeV}}\right]\left[\frac{m_\phi}{10^{13}{\rm GeV}}\right]^{1/2}, \tag{32}$$

Here, let us consider a QCD axion model with right-handed neutrinos [58, 59, 60, 61] and identify ϕ as the field responsible to the SSB of PQ symmetry. When the SSB occurs after inflation, cosmic strings form at the phase transition [62]. After the QCD phase transition, the non-perturbative effect associated with instantons breaks $U(1)_{\rm PQ}$ symmetry down to Z_n, where n is an integer depending on models. This implies that domain walls form at the QCD phase transition. While these domain walls are short lived in the case of $n = 1$ [63, 64, 65, 66], they are stable and disastrous in the case of $n \geq 2$ [19, 20]. One of the simplest solution of this domain wall problem is that the PQ symmetry is never restored after inflation.♠16 This scenario requires a reheating temperature lower than the one derived in Eq. (4.2). QCD axion models predict a pseudo-NG boson called axion, which is a good candidate of DM. The abundance of axion is related to the PQ breaking scale v, so that the observed DM abundance determines its value [67, 68, 69]. The result is given as

$$v \simeq 8 \times 10^{11}\ {\rm GeV} \times n\left|\theta_0\right|^2, \tag{4.3}$$

where θ_0 is the initial phase of axion field and n is the domain wall number. This implies that the reheating temperature should be lower than 10^{10} GeV [see Eq. (4.2)]. Hence, leptogenesis may be marginally realized to explain the baryon asymmetry of the Universe [70] (see also Ref. [71]).

taken from arXiv:1506.07661