Thermalization in gauge theories

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Based on:

- Kurkela, Moore (2011) [arXiv:1107.5050]
- Schlichting, Teaney (2019) [arXiv:1908.02113]
- Kurkela, Lu (2014) [arXiv:1405.6318]

Outline

- Motivation
- Setup
- QCD kinetics
- Thermalization
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  - underoccupied systems
- Summary

Motivation

Relativistic out-of-equilibrium plasmas can occur e.g.

- in relativistic heavy-ion collisions,
- after inflation in the early universe.

The subsequent thermalization can be a highly non-trivial process, depending on initial conditions, dominant processes governing particle dynamics, particle content,...

- How do such plasmas thermalize?
- What time and momentum scales enter?

Setup

We consider closed systems within a non-abelian gauge theory which are

- isotropic and homogeneous,
- weakly coupled,
- either close to or far from equilibrium.

Example: QCD; plasma consisting of quarks and gluons at high temperatures.

$\rightarrow$ Use parametric estimates to describe dynamics and equilibration mechanism of the Quark-Gluon-Plasma.

QCD Kinetics

Idea: On sufficiently long time and distance scales, dynamics of the underlying QFT can effectively be described by kinetic theory.

Dynamics of typical ultra-relativistic excitations given via a Boltzmann eq.

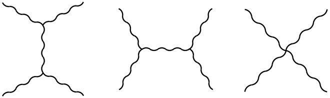
$$(\partial_t + \mathbf{v}_p \cdot \partial_{\mathbf{x}})f = -C[f] \tag{1}$$

with $f = f(x, p, t)$.
For QCD kinetic theory, two different contributions to $C[f]$ at leading order in the coupling:

$$C[f] = C_{\text{el}}^{2\leftrightarrow 2}[f] + C_{\text{inel}}^{1\leftrightarrow 2}[f]. \tag{2}$$

QCD Kinetics

$C_{\mathrm{el}}^{2\leftrightarrow 2}[f]$:



Matrix element for elastic scattering contributions dominated by small momentum exchange $q_\perp$,

$$|M|^2 \sim \frac{\alpha^2}{q_\perp^2(q_\perp^2 + m^2)} \tag{3}$$

where $m^2 \sim \alpha \int_p f(p)/p$. m is referred to as the *screening scale* and appears as effective mass in particle dispersion at leading order,

$$E(p) = \sqrt{p^2 + m^2}. \tag{4}$$

QCD Kinetics

Elastic scatterings with small $q_\perp$ lead to momentum diffusion, with average squared momentum transfer growing linearly in time,

$$\Delta p^2 \sim \hat{q}_{\rm el} t. \tag{5}$$

The diffusion constant $\hat{q}_{\rm el}$ is estimated from the elastic-collision rate

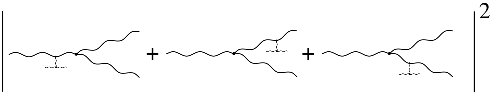
$$\Gamma_{\rm el} \sim \frac{\alpha^2}{m^2} \int_p f(p)[1 + f(p)] \sim \frac{\hat{q}_{\rm el}}{m^2}, \tag{6}$$

giving

$$\hat{q}_{\rm el} \sim \alpha^2 \int_p f(p)[1 + f(p)]. \tag{7}$$

QCD Kinetics

$C_{\mathrm{inel}}^{1\leftrightarrow 2}[f]$:



Soft elastic scatterings in a medium induce nearly collinear splitting processes.

A splitting process is completed on a time-scale t_{form}, when the wave packets of emitter and emitted particle fully separate in the transverse direction.

If $t_{\mathrm{form}} \ll \Gamma_{\mathrm{el}}^{-1}$: Splitting rate given by Bethe and Heitler

$$\frac{d\Gamma_{\mathrm{split}}^{\mathrm{BH}}}{dp/p} \sim \alpha \Gamma_{\mathrm{el}}, \tag{8}$$

where interferences between scattering events can be neglected.

If $t_{\text{form}} \gtrsim \Gamma_{\text{el}}^{-1}$: *LPM effect*: Emitted particle stays on top of emitter and interferes with emitted particle from next scattering.

Emitter experiences net deflection from all collisions,

$$\Gamma_{\text{split}}^{\text{LPM}} \sim \alpha t_{\text{form}}^{-1} \quad \text{with} \quad t_{\text{form}}(k) \sim \sqrt{\frac{k}{\hat{q}_{\text{el}}}}. \tag{9}$$

For high-energetic particles, formation time can be long compared to time between elastic collisions
$\rightarrow$ suppression of radiative emission at high energies.

We denote the momentum scale at which the spectrum of split particles changes from BH to LPM type by k_{LPM},

$$t_{\text{form}}(k_{\text{LPM}}) \sim \Gamma_{\text{el}}^{-1}. \tag{10}$$

Thermalization

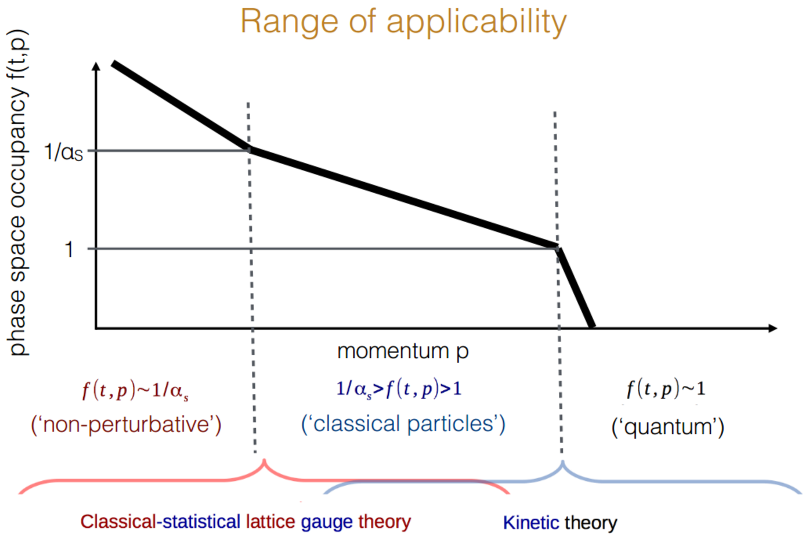
Let Q be the momentum of a typical quasi-particle excitation in the plasma. The occupancy can be written parametrically as

$$f(Q) \sim \alpha^{-c}, \tag{11}$$

which allows to distinguish between:

- $0 < c < 1$: overoccupied system
- $c < 0$: underoccupied system
- $c > 1$: extremely overoccupied system; non-perturbative regime

Thermalization



taken from
http://www.int.washington.edu/talks/WorkShops/int_15_2c/People/Schlichting_S/Schlichting.pdf

Thermalization

Overoccupied system

Energy density initially dominated by large number of low-momentum modes,

$$f(p \lesssim Q) \sim \alpha^{-c} \quad \text{with} \quad 0 < c < 1 \tag{12}$$

giving

$$e \sim \int d^3p\, p\, f(p) \sim \alpha^{-c} Q^4. \tag{13}$$

In equilibrium $e \sim T^4$. Energy conservation gives final temperature

$$T \sim \alpha^{-c/4} Q \gg Q. \tag{14}$$

The subsequent thermalization is driven by elastic scatterings and merging processes, which will have parametrically identical rates.

Thermalization

With $t \sim p^2/\hat{q}_{\rm el}$ and $\hat{q}_{\rm el} \sim \alpha^2 \int_p f(p)[1+f(p)]$, we find that out of all low momentum modes, the lowest ones have the smallest scattering time scale.

Before thermalization, system forgets about initial conditions as the lowest modes quickly enter quasi-equilibrium state, also referred to as the *non-thermal fixed point*.

In this regime, occupancies have a self-similar scaling form

$$f(t,p) = t^a f_S(t^b p) \tag{15}$$

with scaling exponents $a = -4/7$ and $b = -1/7$ determined from energy conservation and the scaling behavior of the Boltzmann eq.

Thermalization

Thermalization is achieved by energy cascading to higher momentum modes with lower occupancies until momenta $\sim T$ and occupancies ~ 1.

For estimating the equilibration time, we model the modes below some $p_{\max}$ as quasi-equilibrated with scale T_*,

$$f(p) \sim \frac{T_*}{p}\Theta(p_{\max} - p), \tag{16}$$

giving energy density

$$e \sim T_* p_{\max}^3. \tag{17}$$

Initially $p_{\max} < T < T_*$. The cascade lets $p_{\max}$ grow in time, T_* will decrease due to energy conservation.

Thermalization

Momentum diffusion due to elastic scatterings:

$$\hat{q} \sim \alpha^2 \int_p f(p)[1+f(p)] \sim \alpha^2 T_*^2 p_{\rm max}. \qquad (18)$$

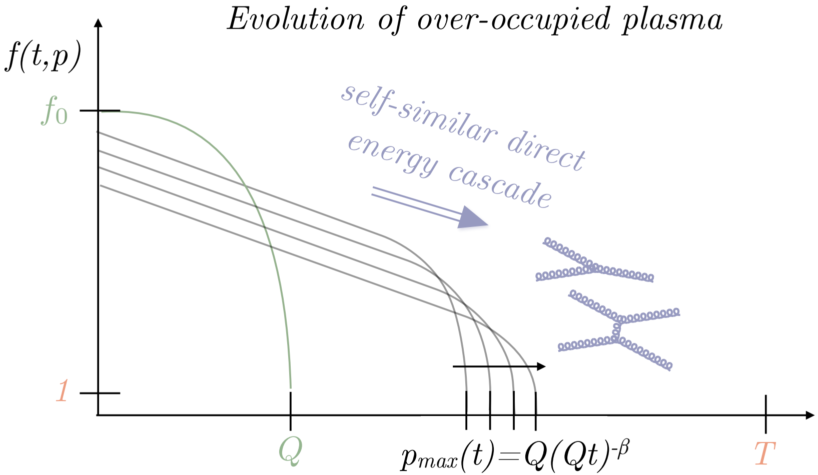
Together with $p_{\rm max}^2 \sim \hat{q}t$ and energy conservation one finds time evolution

$$p_{\rm max} \sim \alpha^{\frac{2-2c}{7}} Q^{\frac{8}{7}} t^{\frac{1}{7}}, \quad T_* \sim \alpha^{\frac{-6-c}{7}} Q^{\frac{4}{7}} t^{-\frac{3}{7}}. \qquad (19)$$

Thermalization achieved when $p_{\rm max} \sim T_*$, i.e., at time

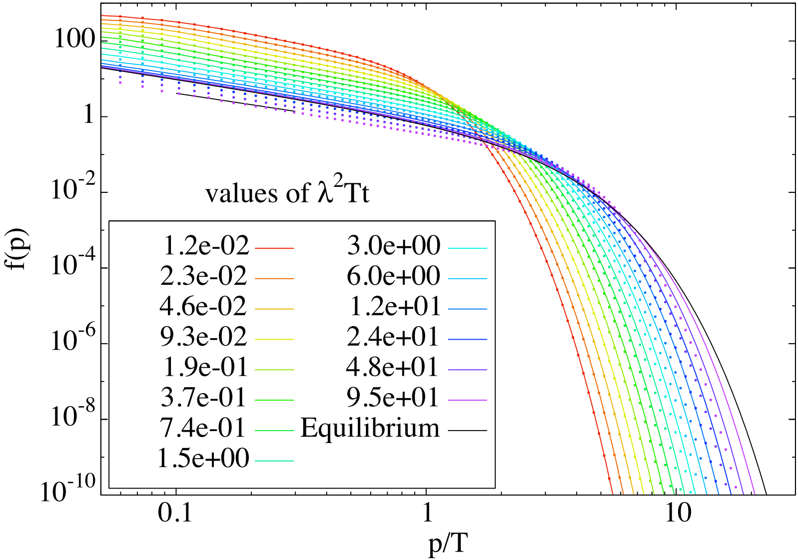
$$t \sim \alpha^{-2+c/4} Q^{-1} \sim \alpha^{-2} T^{-1} \equiv t_{\rm eq}. \qquad (20)$$

Thermalization



Schlichting, Teaney (2019)

Thermalization



Kurkela, Lu (2014)

Underoccupied system

Energy density initially dominated by small number of high-momentum modes,

$$f(p \lesssim Q) \sim \alpha^{-c} \quad \text{with} \quad c < 0, \tag{21}$$

where now $T_{\text{final}} < Q$.

The "bottom-up" thermalization process can be divided into three phases.

Thermalization

First phase:

Emission of soft radiation leads to a population of low momentum modes. They can again be characterized by the scales $p_{\max}$ and T_* via

$$f(p) \sim \frac{T_*}{p}\Theta(p_{\max} - p). \tag{22}$$

The growth of $p_{\max}$ is mainly driven by elastic scatterings with the hard modes, giving

$$p_{\max} \sim \alpha^{1-\frac{c}{2}} Q(Qt)^{\frac{1}{2}}. \tag{23}$$

Thermalization

Second phase:

Emission of soft radiation gets LPM suppressed as $p_{\rm max} \sim k_{\rm LPM}$. The effective temperature T_* of the soft sector starts decreasing over time,

$$T_* \sim \alpha^{-\frac{1}{2}-\frac{c}{4}} Q (Qt)^{-\frac{1}{4}}. \tag{24}$$

Soft sector thermalizes when $p_{\rm max} \sim T_*$, i.e., for

$$t \sim \alpha^{-2+c/3} Q^{-1}. \tag{25}$$

At that time the soft modes carry only a small portion of the energy density.

Thermalization

Third phase:

Radiative branching of high energy modes leads to inverse energy cascade.
Let p_{split} be the scale at which the probability for a single hard mode to undergo multiple splittings in time t becomes order 1,

$$\Gamma_{\text{split}}^{\text{LPM}}(p_{\text{split}})t \sim 1. \tag{26}$$

With T being the single scale describing the soft thermal bath, one finds

$$p_{\text{split}} \sim \alpha^2 T^3 t^2. \tag{27}$$

At early times this scale is very low, energy transfer from hard splittings to the soft bath is negligible.

Thermalization

Using energy conservation, one finds time evolution

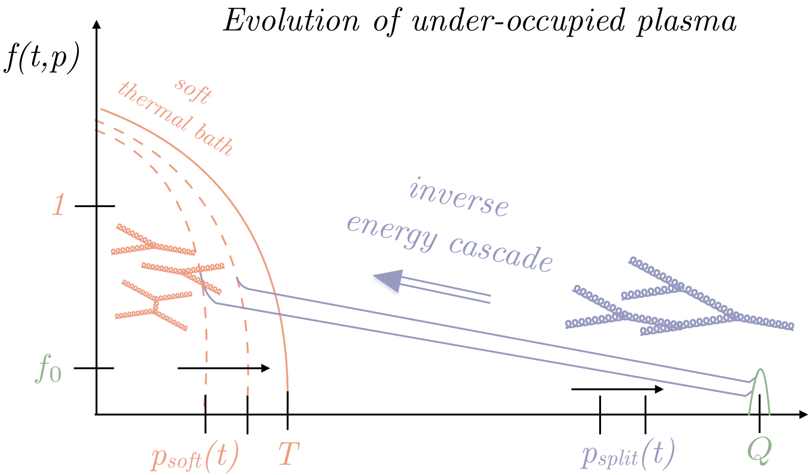
$$T \sim \alpha^{4-c} Q^3 t^2, \tag{28}$$

$$p_{\text{split}} \sim \alpha^{16-3c} Q (Qt)^8. \tag{29}$$

When $p_{\text{split}} \sim Q$, all hard modes had enough time to split and join the thermal bath.

Equilibration occurs when $T \sim T_{\text{final}} \sim \alpha^{-\frac{c}{4}} Q$, which happens at the time

$$t \sim \alpha^{-2} T^{-1} \sqrt{\frac{Q}{T}} \equiv t_{\text{eq}}. \tag{30}$$



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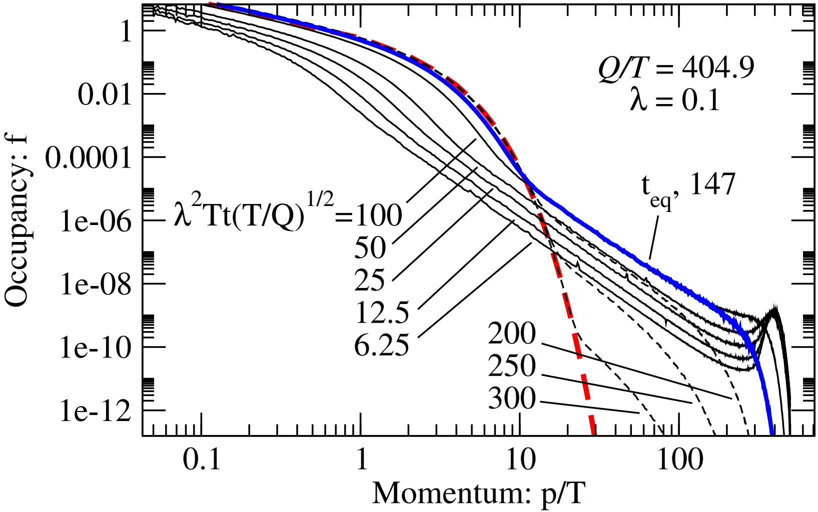

Kurkela, Lu (2014)

Summary

- Using kinetic theory we could identify the relevant physics driving the general equilibration process of a non-abelian plasma
- Overoccupied systems equilibrate via energy cascade on a time scale $t_{\rm eq} \sim \alpha^{-2} T^{-1}$
- Underoccupied systems equilibrate by establishing a soft thermal bath, followed by hard mode splittings, $t_{\rm eq} \sim \alpha^{-2} T^{-1} \sqrt{\frac{Q}{T}}$

Related topics not covered here:

- anisotropic systems, jet physics; relevant in heavy-ion collisions
- extreme overoccupancies (Nielsen-Olesen instability)