Inflaton's energy transfer via (p)reheating

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Based on:

M. A. Amin et al. "Nonperturbative dynamics of reheating after inflation: a review", arXiv:1410.3808

K. Lozanov, "Lectures on Reheating after Inflation", arXiv:1907.04402

End of Inflation…

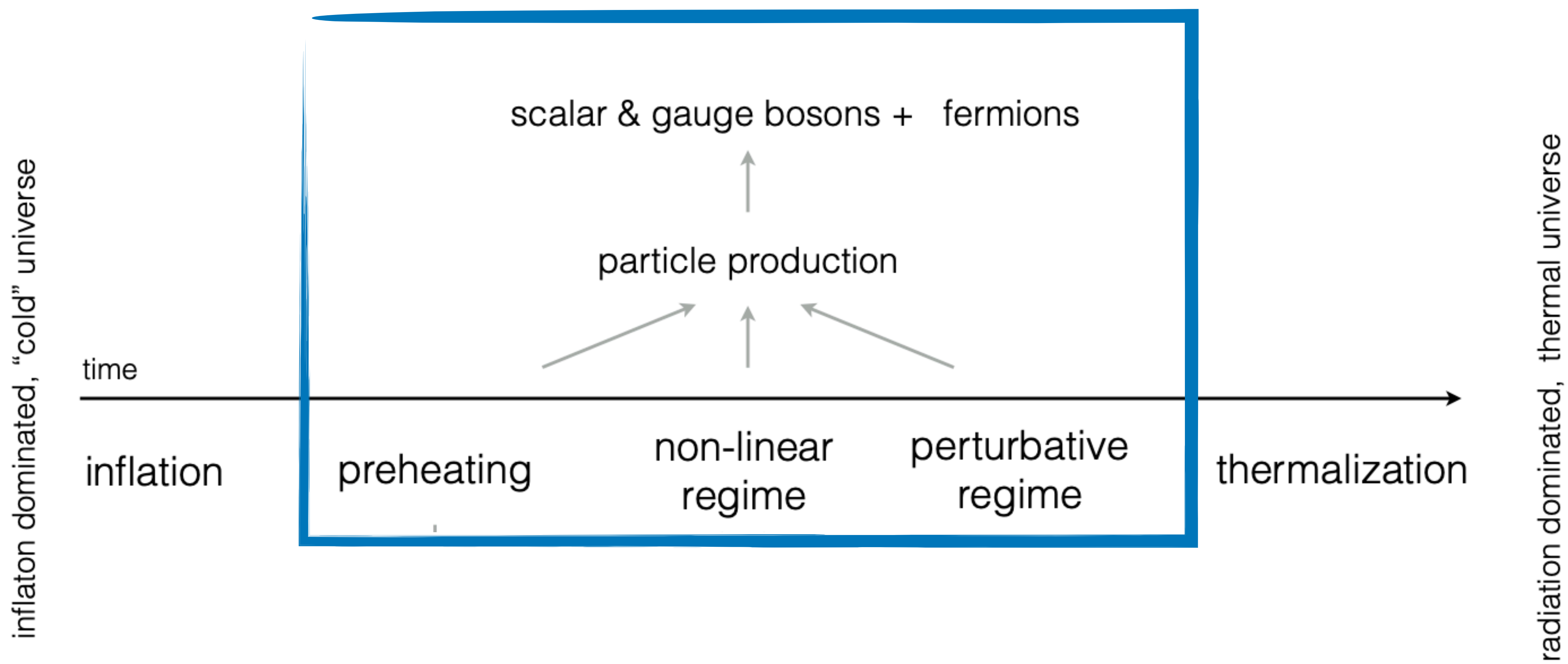
- Inflation ends when slow roll conditions are violated…fields begin to fall towards the minimum of potential.
- Using Friedman and Euler-Lagrange equation,

$$H \approx \frac{2}{3t}\left(1+\frac{\sin(2mt)}{2mt}\right), \qquad \text{and} \qquad \bar{\phi} \approx \frac{2\sqrt{2}m_{\rm Pl}\cos(mt)}{\sqrt{3}mt}\left(1+\frac{\sin(2mt)}{2mt}\right)$$

- At the end of inflation, nothing but decaying inflationary scalar field condensate.
- How does the matter of which we are made up of arises?
- How universe is reheated?
- Initial energy density damped via expansion and "*particle production*".

Outline

At the end of inflation, inflaton field must decay into other forms of matter and radiation.



Reheating: Perturbative Approach

$$S_{int} \supset \int d^4x\sqrt{-g}(-\sigma\phi\chi^2 - h\phi\bar{\psi}\psi)$$

χ and ψ are some scalar and fermion product.

At tree level, the decay rates are given as

$$\Gamma_{\phi\to\chi\chi} = \frac{\sigma^2}{8\pi m}, \qquad \Gamma_{\phi\to\bar{\psi}\psi} = \frac{h^2 m}{8\pi}.$$

which determines the decay rate of number of inflaton quanta in a fixed comoving volume

$$\frac{d\left(a^3 n_{\bar{\phi}}\right)}{dt} = -\Gamma_{\mathrm{tot}} a^3 n_{\bar{\phi}}$$

If $\Gamma_{tot}^{-1} >> m^{-1}$ then,

$$\bar{\phi} \sim \mathrm{a}^{-3/2}(\mathrm{t})\exp(-\Gamma_{\mathrm{tot}}\mathrm{t}/2)$$

Reheating: Perturbative Approach

CRC-TR 211

For $\Gamma_{tot} << H$ Energy loss due to expansion.

$H \leq \Gamma_{tot}$ Particle production becomes effective

Since $\Gamma \propto \sigma^2$, the perturbative decay is quite small. Leading to more e-folds of expansion after inflation in order to have $H \leq \Gamma_{tot}$. 👎

Bose condensation effect not taken into account. 👎

$$\frac{d\left(a^3 n_\chi\right)}{dt} = \frac{2a^3}{V_{\mathrm{com}}}\Gamma_{\phi\to\chi\chi}\left[(n^\chi_{\mathbf{k}}+1)(n^\chi_{-\mathbf{k}}+1)n^\phi_{\mathbf{0}} - n^\chi_{\mathbf{k}}n^\chi_{-\mathbf{k}}(n^\phi_{\mathbf{0}}+1)\right]$$

$$\approx 2a^3\Gamma_{\phi\to\chi\chi}n_{\bar{\phi}}\left[1+2n^\chi_k\right] \approx 2a^3\Gamma_{\phi\to\chi\chi}n_{\bar{\phi}}\left[1+\frac{2\pi^2\bar{\Phi}}{\sigma}\frac{n_\chi}{n_{\bar{\phi}}}\right]$$

Fails for large coupling constant. 👎

Coherent nature of inflaton condensate also suggests that non perturbative effects should be taken into consideration.

Parametric Resonance

Assuming Bose effect becomes more effective, the occupation number of particles χ is given as:

$$n_\chi \sim \exp\left(\frac{\pi\sigma\bar{\Phi}t}{2m}\right)$$

- Bose condensation effects can exponentially enhance the rate of energy transfer from the inflaton to the coupled bosonic fields.
- Treating matter fields as fluctuations on top of oscillating homogenous background inflaton field.
- For a “trilinear interaction model” (TIM), with potential as

$$V(\phi,\chi) = m^2\phi^2/2 + m_\chi^2\chi^2/2 + \sigma\phi\chi^2$$

the EoM is

$$\ddot{\hat{\chi}}_{\mathbf{k}} + \omega^2(k,t)\hat{\chi}_{\mathbf{k}}(t) = 0 \qquad (1)$$

with periodic angular frequency $\omega^2(k,t) = k^2 + m_\chi^2 + 2\sigma\bar{\phi}\cos(\mathrm{mt})$

Parametric Resonance

1 can be reduced to Mathieu equation.

$$\frac{d^2}{dz^2}\hat{\chi}_{\mathbf{k}} + \left[A_k + 2q\cos(2z)\right]\hat{\chi}_{\mathbf{k}}(z) = 0$$

with

$$A_k = 4(k^2 + m_\chi^2)/m^2 \qquad q = 4\sigma\bar{\phi}/m^2 \qquad z = mt/2$$

The mode function of $\hat{\chi}_k$ satisfies the equation

$$\ddot{u}_k + \omega^2(k,t)u_k(t) = 0$$

The Floquet theorem gives the most general solution for the mode function of $\hat{\chi}_k$ as:

$$u_k(t) = e^{\mu_k t}\mathcal{P}_{k+}(t) + e^{-\mu_k t}\mathcal{P}_{k-}(t)$$

where μ_k is called **Floquet exponent** and $\mathcal{P}_{k\pm}(t) = \mathcal{P}_{k\pm}(t+T)$

If $\Re(\mu_k) \neq 0$ one of the two terms increases exponentially with time. This is called **parametric resonance**.

If $u_k(t)$ is a solution, then $u_k(t+T)$ is also a solution. Taking $u_{k1}(t)$ and $u_{k2}(t)$ be two linearly independent solution,

$$u_{ki}(t+T) = \sum_{j=1}^{2} B_{ij}u_{kj}(t)$$

Parametric Resonance

$$u_{ki}(t+T) = \sum_{j=1}^{2} B_{ij} u_{kj}(t)$$

where B_{ij} is 2x2 invertible matrix. Get the eigenvalues as $\lambda_{1,2}^{B}$ to calculate the Floquet exponent:

$$\mu_k = \frac{\ln(\lambda^{\mathrm{B}})}{T}$$

For a particular choice of initial conditions

$$\{u_{k1}(t_0), \dot{u}_{k1}(t_0)\} = \{1, 0\} \text{ and } \{u_{k2}(t_0), \dot{u}_{k2}(t_0)\} = \{0, 1\}$$

By evolving the Mathieu's equation for one period T, the Eigen values are given as

$$\lambda_{1,2}^{B} = \frac{1}{2}\{u_{k1}(t_0+T) + \dot{u}_{k2}(t_0+T) \pm \sqrt{[u_{k1}(t_0+T) - \dot{u}_{k2}(t_0+T)]^2 + 4\dot{u}_{k1}(t_0+T)u_{k2}(t_0+T)}\}$$

Vacuum fluctuation plays a crucial role for particle production.

Instability chart: Narrow Resonance

Instability chart for Mathieu equation

$$\frac{d^2}{dz^2}\hat{\chi}_{\mathbf{k}} + \left[A_k + 2q\cos(2z)\right]\hat{\chi}_{\mathbf{k}}(z) = 0$$

- Narrow Resonance for:

$$|q| \ll 1 \quad \text{and} \quad \mathrm{A_k} > 0$$

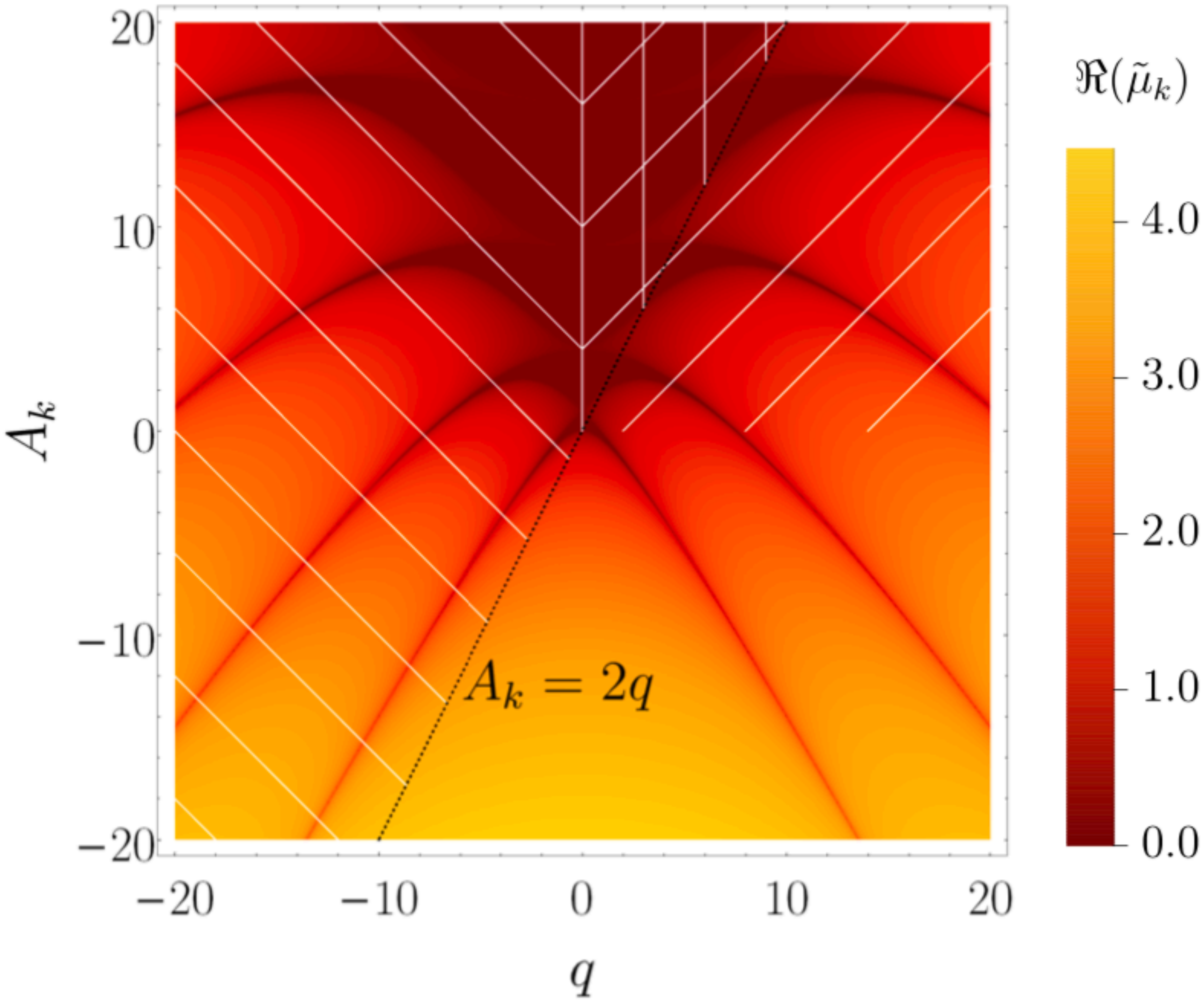
- Resonant production of χ particles.
- In first narrow band:

$$\Re\left(\tilde{\mu}_k\right)^{(1)}_{\max} \approx |q|/2, \text{ while } A_k^{(1)} \approx 1 \pm |q|$$

- Occupation number grows as

$$\sim \exp\left(\frac{2\sigma\bar{\phi}\mathrm{t}}{\mathrm{m}}\right)$$

- Bose effect described at first n=1, narrow, q<<1 instability band.
- Perturbative limit.

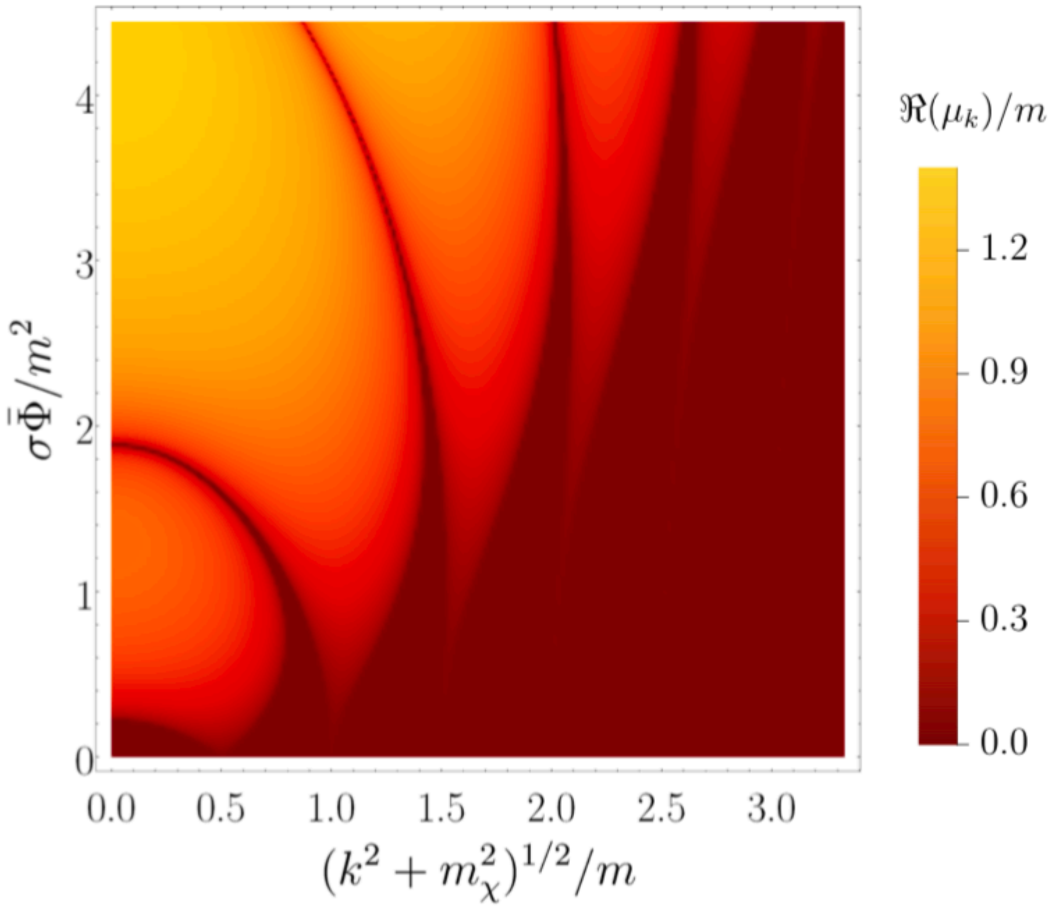


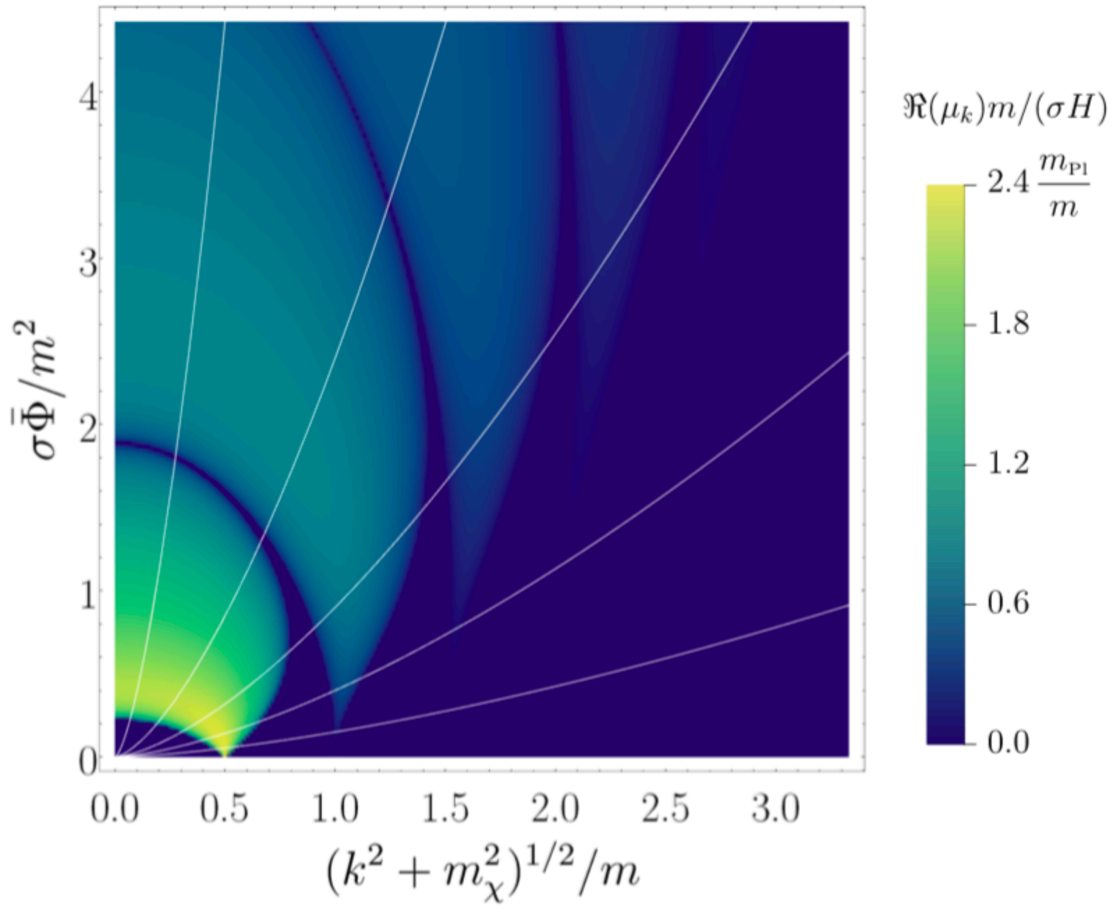
Instability chart: Broad Resonance

- Occurs for $1 \leq |q|$ in Mathieu equation.
- Corresponds to non perturbative limit in TIM.
- Particles produced in burst, when adiabadicity condition is violated.

$$\frac{\dot{\omega}(k,t)}{\omega^2(k,t)} \ll 1 \qquad\qquad \omega(k,t) = A_k + 2q\cos(2z)$$

- Like if, $A_k \leq 2|q|$ the inequality is not satisfied near $z_j = \pi/4,\, 3\pi/4, ..$





Stochastic Resonance: Expansion of Space

- Taking Background space-time curvature into account.
- Equation of motion for scalar fields can still be reduced to SHO

  using $\hat{\chi}_c(t) = a(t)^{3/2}\hat{\chi}(t)$ t is cosmic time

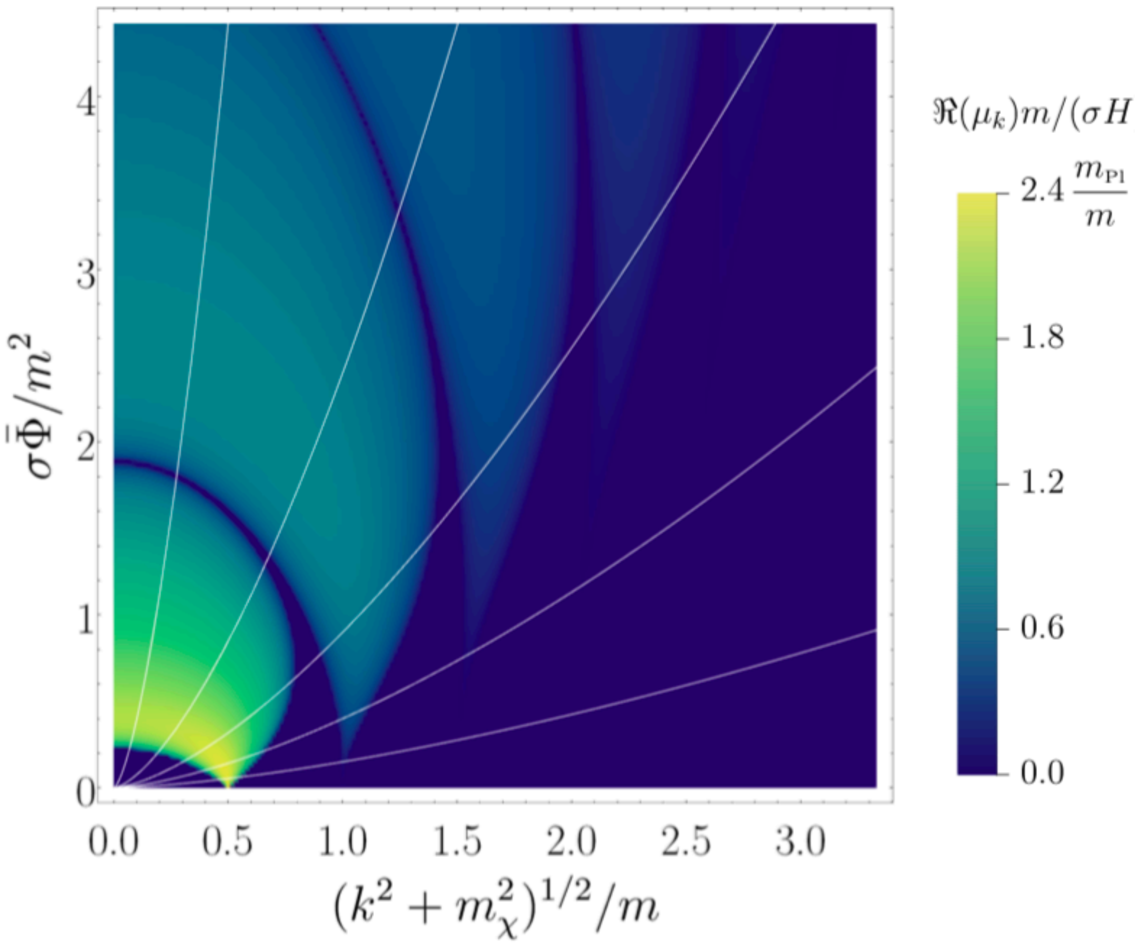
$$\ddot{\hat{\chi}}_{c\mathbf{k}} + \omega^2(k,t)\hat{\chi}_{c\mathbf{k}}(t) = 0$$

$$\omega^2(k,t) = \left(\frac{k}{a}\right)^2 + m_\chi^2 + 2\sigma\bar{\phi}(t)\cos(\mathrm{mt}) - \left(\frac{3\mathrm{H}}{2}\right)^2 - \frac{3}{2}\dot{\mathrm{H}}$$

- Not a Hill's equation.
- Qualitative understanding using

$$\bar{\phi} \propto a^{-3/2} \quad k \propto a^{-1}$$

- Co-moving waves redshifted.



Stochastic Resonance: Expansion of Space

- Condition for parametric resonance to result in significant particle production is

$$\frac{|\Re(\mu_k)|}{H} \gg 1$$

- Broad resonance enhance or shut off…
- Floquet index

$$\mu_k^j = \ln \left| \frac{1 + |R_k^j| e^{i(\theta_k^j + \Delta\theta_k^j)}}{\sqrt{1 - \left|R_k^j\right|^2}} \right| .$$

- Floquet index can change stochastically b/w successive particle creation events, hence broad resonance for an expanding space is called **stochastic resonance.**
- Narrow Resonance becomes kinda sensitive…

Multi Field Preheating:

- Periodicity condition can be violated.
- Can still violate the adiabaticity condition.
- Randomness due to reflection coefficient, phase and oscillation time.
- Analogy based on Condensed matter phenomenon of **Anderson Localisation,** that effective masses with random component give rise to exponentially growing modes.
- Realising a strict periodic condition at the end of inflation is difficult.
- Always a quasi periodic motion at the background level.

Stochastic Resonance: Tachyonic Instabilities

- Effective frequency $\omega^2(k,t)$ changes periodically with time.
- Hybrid inflation

$$V(\phi,\chi) = \lambda_\chi(\chi^2 - v^2)^2 + g^2\phi^2\chi^2 + V(\phi)$$

- As $\phi < \lambda_\chi v^2/g^2$, the sign of frequency changes from +ve to -ve value for long wavelength modes.
- Would still lead to particle production.
- A negative squared frequency=> “imaginary mass”, hence the term “tachyonic preheating”
- Occurs in symmetry breaking models.
- In order to have efficient Tachyonic decay of inflaton condensate

$$\frac{|m_{\chi,\mathrm{eff}}|}{H} \gg 1$$

Instant preheating

- Arises due to time dependent effective mass of the fluctuation.
- For a coupling of χ to some fermion ψ of Yukawa form $h\chi\psi\bar{\psi}$

  $\chi \to \psi\bar{\psi}$ forbidden if $m_\chi < 2m_\psi$
- Kinematically allowed if scalar is coupled to a inflaton via $g^2\phi^2\chi^2/2$

$$\Gamma_{\chi\to\bar{\psi}\psi} = \frac{h_\chi^2 g|\bar{\phi}|}{8\pi}$$

- Between two consecutive particle production (adiabaticity condition violated for $\bar{\phi} \sim 0$), $\Gamma_{\chi\to\bar{\psi}\psi}$ maximal.
- If certain χ particles are produced, they can all decay into fermions. This mechanism is called **instant preheating.**

Gauge Fields

- If inflaton is a gauge singlet, possible coupling:

$$S_{\mathrm{matter}} \supset \int d^4x\sqrt{-g}\,[-W_1(\phi)F_{\mu\nu}F^{\mu\nu} - W_2(\phi)\epsilon^{\mu\nu\eta\sigma}F_{\mu\nu}F_{\eta\sigma}]$$

- First term: transfer of energy to massless gauge bosons.
- Second term:
  - Violates parity
  - Arises in model where inflaton is an axion.
- If inflaton is charged under a gauge symmetry (*complex scalar, charged under Abelian U(1) symmetry*)

$$S_{\mathrm{matter}} \supset \int d^4x\sqrt{-g}\,D_\mu\phi(D^\mu\phi)^* = \int d^4x\sqrt{-g}\left[\partial_\mu\phi\partial^\mu\phi^* + \boxed{2g_A\,\Im(\phi\partial_\mu\phi^*)A^\mu} + g_A^2|\phi|^2A_\mu A^\mu\right]$$

- Transverse component of gauge field behave like χ
- Recent studied shows, available approximations are insufficient…

In a nutshell…

Scalar fields:

- Particle production understood using Floquet theory. Exponential growing occupation number based on the interaction.
- Stochastic resonance arises when background expansion of space and multi-fields are included.

Fermions:

- Interaction of Yukawa form allows for inflaton to decay into fermion product.
- Important process to have radiation dominated universe.
- From instant preheating.

Thank you…